Two-Way Relaying with Multiple Antennas using Covariance Feedback

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***Abstract*—This paper studies two-way multi-antenna amplify-and-forward relaying using physical layer network coding. So far, the design of the relay matrix for sum rate maximization has been based on the assumption of complete channel state information (CSI). Designs for the relay matrix based on covariance feedback are proposed in this paper. The optimal relay matrix is estimated using the Monte Carlo method, in which a large number of channel realizations are generated. This requires a runtime linear in the number of channel samples used. Low-complexity methods are then proposed to calculate the relay matrix, without the need to generate multiple channel realizations. Simulations show that the low-complexity methods proposed have near-optimal performance. These methods exhibit much larger sum rates than direct relaying for small to moderate angular spreads. Furthermore, as the number of antennas increases, the low-complexity methods show significant improvements over direct relaying.**

I. Introduction

Relaying is an important technique that allows long distance wireless communication between terminals. Within a cellular system, a relay could be used between a base station and a user terminal, thereby increasing the transmission range. Wireless relays can also be used between base stations as a cost-effective alternative to cable links. Relays are also important building blocks in wireless ad hoc networks, which cut down the power consumption of the terminals, consequently improving the network lifetime.

In bidirectional or two-way communication, information is to be exchanged between two terminals. Recently, there has been great interest in the application of network coding [1] to the wireless fading channel. This has been known as physical layer network coding (PNC) [2, 4] or analogue network coding [3]. In PNC, the senders transmit at the same time and a superposition of the electromagnetic waves arrive at the relay node. The relay then simply performs amplify-and-forward (AF) or non-regenerative relaying. Consider a setup in which two terminals, T1 and T2, exchange information via a relay terminal RL over two phases. In the first phase, T1 and T2 transmit and RL receives. In the second phase, RL transmits, while T1 and T2 receive. If T1 has complete channel state information (CSI) of all the links, it is able to cancel out its own self-interference and decode the information from T2, since it already knows its own signal. The same is true for T2.

Apart from AF, the decode-and-forward (DF) protocol has been considered in [6] for the two-way relay. It has been shown that when RL is equidistant from T1 and T2, the AF scheme can perform better than the DF [7]. The reason is that the DF scheme has to cope with a multiple access channel in the first phase, which achieves the highest sum rate for an asymmetric channel, in which one channel is much stronger than the other.

The use of multiple antennas improves rate and link reliability without requiring an increase in transmit power. [5] proposes optimal and suboptimal designs for the relay matrix of a multi-antenna two-way relay given instantaneous CSI of all the channels.

In the examples mentioned above, the relay matrix is designed based on instantaneous CSI of all the links. Covariance feedback, however, is a favourable option as the period over which statistical CSI is valid is much longer than the duration of each fade. For the case of time division duplex (TDD), channel reciprocity can be used to obtain the covariance CSI implicitly. Even for the case of frequency division duplex (FDD), the frequency calibration technique [13] is able to estimate the transmit channel covariance matrix based on the receive channel covariance matrix, without the use of feedback.

Space-time transmit precoding based on covariance feedback was first examined in [8] in which it was shown that the eigenvectors of the optimal source covariance matrix are exactly the eigenvectors of the transmit channel covariance matrix, for point-to-point multiple-input single-output (MISO) communication.

While the calculation of the optimal source covariance eigenvalues is a challenging task, there has been recent advances [9–11]. Estimation of the optimal eigenvalues require Monte Carlo approximation which has a runtime linear in the number of channel samples (i.e. independent channel realizations). A suboptimal solution, in which the source covariance eigenvalues are generated by water-filling over the channel covariance eigenvalues [9], allows a simple approximation without performing the Monte Carlo trials. In relay channels, the design of the relay matrix for the one-way multi-antenna AF scheme using covariance feedback was first proposed in [12]. To date, there has been no work on two-way multi-antenna AF relaying based on covariance feedback. This paper proposes both high- and low-complexity designs for the relay matrix. Computer simulations are used to compare the performance of the various schemes.

Section II outlines the system model and the channel model. Sum rate maximization based on instantaneous CSI is reviewed in Section III. Covariance-based design of the relay matrix is proposed in Section IV. Low-complexity methods for sum rate maximization are then proposed in Section V. Simulation results are given in Section VI, and are followed by the conclusion in Section VII.

Notations:

Vectors and matrices are denoted by boldface letters. $(\cdot)^*$, $(\cdot)^T$, and $(\cdot)^H$ denote the conjugate, transpose, and conjugate transpose operations respectively. $\mathbf{I}_M$ is the $M \times M$ identity matrix. $\mathbb{C}^{M\times N}$ is the set of complex $M \times N$ matrices. For block matrices, $[\mathbf{A};\mathbf{B}]$ represents vertical concatenation i.e. $[\mathbf{A}^T, \mathbf{B}^T]^T$. $\mathbb{E}[\cdot]$ and $\mathrm{Tr}(\cdot)$ are the expectation and trace operators respectively. $|\cdot|$ and $\|\cdot\|$ denote the absolute value of a complex scalar and the vector Euclidean norm, respectively. $\mathcal{CN}(\mathbf{m}, \mathbf{R})$ refers to the multi-variate complex Gaussian distribution with mean $\mathbf{m}$ and covariance $\mathbf{R}$.

II. System Model and Channel Model

This section describes the system model and channel model for the two-way multi-antenna relaying scheme.

A. System Model

The relay terminal RL is equipped with M antennas while terminals T1 and T2 have 1 antenna each. Assume no CSI is available at the transmitters of T1 and T2, while complete CSI is available at the receivers of T1 and T2. Furthermore, only the covariance matrices of each channel are available at RL. Communication is carried out over 2 phases, phase A and phase B. Fig. 1 shows the schematic diagram of the setup. In phase A, T1 transmits $x_1 = \sqrt{P_1}s_1$ where s_1 is the data symbol with $\mathbb{E}[|s_1|^2] = 1$ and P_1 is the transmit power. Likewise, T2 transmits $x_2 = \sqrt{P_2}s_2$. The relay RL receives $\mathbf{y}_3 \in \mathbb{C}^{M\times 1}$, where

$$\mathbf{y}_3 = \mathbf{h}_1 x_1 + \mathbf{h}_2 x_2 + \mathbf{n}_3 \ . \tag{1}$$

$\mathbf{h}_1 \in \mathbb{C}^{M\times 1}$ and $\mathbf{h}_2 \in \mathbb{C}^{M\times 1}$ are the channels from T1 and T2, respectively, to the relay. $\mathbf{n}_3 \in \mathbb{C}^{M\times 1}$ is the receive noise vector with distribution $\mathcal{CN}(\mathbf{0}, N_0\mathbf{I}_M)$. In phase B, RL transmits $\mathbf{x}_3 = \mathbf{A}\mathbf{y}_3$, where $\mathbf{A} \in \mathbb{C}^{M\times M}$ is the relay amplification matrix. The signal received by node T1 is $y_1 \in \mathbb{C}$, where

$$y_1 = \mathbf{h}_3\mathbf{A}(\mathbf{h}_1\sqrt{P_1}s_1 + \mathbf{h}_2\sqrt{P_2}s_2 + \mathbf{n}_3) + n_1 \ . \tag{2}$$

$\mathbf{h}_3 \in \mathbb{C}^{1\times M}$ is the channel from RL to T1 and $n_1 \sim \mathcal{CN}(0, N_0)$ is the received noise. Since T1 already knows the message s_1, it can cancel out its self-interference, giving

$$\tilde{y}_1 = \mathbf{h}_3\mathbf{A}\mathbf{h}_2\sqrt{P_2}s_2 + \mathbf{h}_3\mathbf{A}\mathbf{n}_3 + n_1 \ . \tag{3}$$

Similarly for node T2,

$$y_2 = \mathbf{h}_4\mathbf{x}_3 + n_2 \tag{4}$$

$$\tilde{y}_2 = \mathbf{h}_4\mathbf{A}\mathbf{h}_1\sqrt{P_1}s_1 + \mathbf{h}_4\mathbf{A}\mathbf{n}_3 + n_2 \ . \tag{5}$$

$\mathbf{h}_4 \in \mathbb{C}^{1\times M}$ is the channel from RL to T2 and $n_2 \sim \mathcal{CN}(0, N_0)$ is the received noise. Denote $r_1(\mathbf{A})$ and $r_2(\mathbf{A})$

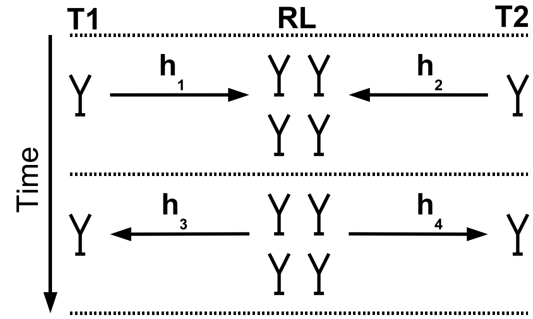

Fig. 1. Schematic Diagram of the Two-Way Relay.

as the information rate of the message originating from nodes T1 and T2 respectively. Then

$$r_1(\mathbf{A}) = \log_2\left(1 + \frac{|\mathbf{h}_4\mathbf{A}\mathbf{h}_1|^2\gamma_1}{\|\mathbf{h}_4\mathbf{A}\|^2 + 1}\right) \quad \text{and} \tag{6}$$

$$r_2(\mathbf{A}) = \log_2\left(1 + \frac{|\mathbf{h}_3\mathbf{A}\mathbf{h}_2|^2\gamma_2}{\|\mathbf{h}_3\mathbf{A}\|^2 + 1}\right) \tag{7}$$

where $\gamma_1 = P_1/N_0$ and $\gamma_2 = P_2/N_0$. The sum rate for the information exchange is

$$r(\mathbf{A}) = 0.5(r_1(\mathbf{A}) + r_2(\mathbf{A})) \tag{8}$$

where the factor 0.5 is due to the use of 2 time slots.

B. Channel Model

In this section, the channel model based on covariance feedback is given. The following are the 4 covariance matrices available at RL, and their eigenvalue decompositions.

$$\mathbf{R}_k = \mathbb{E}[\mathbf{h}_k\mathbf{h}_k^H] = \mathbf{U}_k\mathbf{\Sigma}_k^2\mathbf{U}_k^H, \ k = 1, 2 \tag{9}$$

$$\mathbf{R}_k = \mathbb{E}[\mathbf{h}_k^H\mathbf{h}_k]^T = \mathbf{U}_k\mathbf{\Sigma}_k^2\mathbf{U}_k^H, \ k = 3, 4 \ . \tag{10}$$

The channels are modelled as [14]:

$$\mathbf{h}_k = \mathbf{R}_k^{1/2}\mathbf{h}_{wk}, \ k = 1, 2$$

$$\mathbf{h}_k = \mathbf{h}_{wk}(\mathbf{R}_k^{1/2})^T, \ k = 3, 4 \tag{11}$$

where

$$\mathbf{R}_k^{1/2} = \mathbf{U}_k\mathbf{\Sigma}_k\mathbf{U}_k^H, k = 1, 2, 3, 4 \tag{12}$$

$$\mathbf{h}_{wk} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_M), k = 1, 2, 3, 4 \ . \tag{13}$$

If TDD is used, the covariance matrices for the two phases are the same due to reciprocity, i.e. $\mathbf{R}_1 = \mathbf{R}_3$ and $\mathbf{R}_2 = \mathbf{R}_4$. For FDD, even though this reciprocity does not apply, $\mathbf{R}_3$ and $\mathbf{R}_4$ can still be estimated from $\mathbf{R}_1$ and $\mathbf{R}_2$ respectively, using a technique called frequency calibration [13].

III. Review of Sum Rate Maximization given Instantaneous CSI

This section reviews the problem of maximizing the sum rate if the relay has instantaneous CSI of all the channels. Denote $\gamma_3 = P_3/N_0$, where P_3 is the transmit power of RL.

$$\gamma_3 = \|\mathbf{A}\mathbf{h}_1\|^2\gamma_1 + \|\mathbf{A}\mathbf{h}_2\|^2\gamma_2 + \mathrm{Tr}(\mathbf{A}^H\mathbf{A}) \ . \tag{14}$$

The optimal matrix $\mathbf{A}$ can be found by considering the following problem (P1):

$$\begin{aligned} \underset{\mathbf{A}}{\text{maximize}} \quad & r(\mathbf{A}) = 0.5(r_1(\mathbf{A}) + r_2(\mathbf{A})) \\ \text{subject to} \quad & \gamma_3 \le \bar{\gamma}_3 \end{aligned} \tag{15}$$

where $\bar{\gamma}_3$ is the instantaneous transmit power constraint of RL. Let $\mathbf{H}_A = [\mathbf{h}_1, \mathbf{h}_2] \in \mathbb{C}^{M\times 2}$ and $\mathbf{H}_B = [\mathbf{h}_4; \mathbf{h}_3] \in \mathbb{C}^{2\times M}$. Denote the SVDs of $\mathbf{H}_A$ and $\mathbf{H}_B$ as

$$\mathbf{H}_A = \mathbf{U}_A \mathbf{\Lambda}_A \mathbf{V}_A^H \tag{16}$$

$$\mathbf{H}_B = \mathbf{U}_B \mathbf{\Lambda}_B \mathbf{V}_B^H \ . \tag{17}$$

where $\mathbf{\Lambda}_A \in \mathbb{C}^{M\times 2}$ and $\mathbf{\Lambda}_B \in \mathbb{C}^{2\times M}$. Let $\mathbf{U}_a$ be the matrix formed by the first 2 columns of $\mathbf{U}_A$ and $\mathbf{V}_b$ be the matrix formed by the first 2 columns of $\mathbf{V}_B$.

For the case where $\mathbf{h}_1 = \mathbf{h}_3^T$ and $\mathbf{h}_2 = \mathbf{h}_4^T$, it has been proven in [5] that the optimal beamforming matrix for Problem P1 has the following structure:

$$\mathbf{A} = \mathbf{V}_b \mathbf{B} \mathbf{U}_a^H \tag{18}$$

where $\mathbf{B} \in \mathbb{C}^{2\times 2}$ is the matrix to be optimized. It is easy to show that for the general case where $\mathbf{h}_1 \neq \mathbf{h}_3^T$ and $\mathbf{h}_2 \neq \mathbf{h}_4^T$, the same structure holds. In other words, instead of optimizing over M^2 complex variables, the optimization can be done over only 4 complex variables.

IV. Sum Rate Maximization given Covariance CSI

In this section, the design of the relay matrix for rate maximization is considered for the case of covariance feedback.

The optimization problem can be stated as (P2):

$$\begin{aligned} \underset{\mathbf{A}}{\text{maximize}} \quad & \mathbb{E}[r(\mathbf{A})] = \mathbb{E}\left[0.5(r_1(\mathbf{A}) + r_2(\mathbf{A}))\right] \\ \text{subject to} \quad & \mathbb{E}[\gamma_3] \le \bar{p}_3 \end{aligned} \tag{19}$$

where $\bar{p}_3$ is the average power constraint at RL. While, theoretically, an infinite number of samples is required to calculate the expectation $\mathbb{E}[r(\mathbf{A})]$, Problem P2 can be solved to a reasonable accuracy by the sample average approximation (SAA) method [15], in which $\mathbb{E}[r(\mathbf{A})]$ is estimated by averaging over a large number of independent channel realizations. The optimal $\mathbf{A}^\star$ can be estimated by using an exhaustive search. A general software for solving nonlinear optimization problems with constraints (e.g. Matlab) can also be used, although there may be no guarantee that the global maximum is attained. As the simplifying structure in (18) does not apply (unless possibly in highly correlated scenarios), the search has to be carried out over $\mathbf{A} \in \mathbb{C}^{M\times M}$. The optimization is complete when the matrix $\mathbf{A}$ achieves the highest average sum rate $\mathbb{E}[r(\mathbf{A})]$. The following algorithm to solve the average sum rate maximization is proposed.

1. Generate N independent sets of channel realizations $\{\mathbf{h}_1, \mathbf{h}_2, \mathbf{h}_3, \mathbf{h}_4\}$ based on (11).
2. Specify initial value of $\mathbf{A}$.
3. Define the objective function $\hat{r}_N(\mathbf{A})$ where $\hat{r}_N(\mathbf{A}) = \frac{1}{N}\sum_{i=1}^{N} r^i(\mathbf{A})$ and $r^i(\mathbf{A}) = 0.5(r_1(\mathbf{A}) + r_2(\mathbf{A}))$ using (6) and (7) for the channel realization set i.
4. Specify constraint equation $\mathrm{Tr}[(\gamma_1 \mathbf{R}_1 + \gamma_2 \mathbf{R}_2 + \mathbf{I}_M)\mathbf{A}^H \mathbf{A}] \le \bar{p}_3$.
5. Apply the optimization function on the objective given in step 3. and the constraint specified in step 4. to obtain the solution $\hat{\mathbf{A}}_N$.

After obtaining $\hat{\mathbf{A}}_N$, $\hat{c}_N(\hat{\mathbf{A}}_N) = \frac{1}{N}\sum_{i=1}^{N} c^i(\hat{\mathbf{A}}_N)$ is calculated, where $c^i(\hat{\mathbf{A}}_N) = 0.5(r_1(\hat{\mathbf{A}}_N) + r_2(\hat{\mathbf{A}}_N))$ for the channel realization i, and the channel realizations are different from those used in the optimization algorithm, but drawn from the same distribution.

$\hat{r}_N(\hat{\mathbf{A}}_N)$ is an overestimate of $\mathbb{E}[r(\mathbf{A}^\star)]$, while $\hat{c}_N(\hat{\mathbf{A}}_N)$ is underestimate of $\mathbb{E}[r(\mathbf{A}^\star)]$. The difference between $\hat{r}_N(\hat{\mathbf{A}}_N)$ and $\hat{c}_N(\hat{\mathbf{A}}_N)$ is an estimate of the optimality gap [15]. N has to be sufficiently large to ensure this gap is small. As N increases to infinity, these two values converge to $\mathbb{E}[r(\mathbf{A}^\star)]$.

V. Low-complexity Solutions based on Covariance CSI

In this section, low-complexity methods for covariance-based rate maximization are proposed. The advantage of these methods is that Monte Carlo approximation is not required to obtain the matrix $\mathbf{A}$. This has immense implication for hardware design as the processing time is reduced by at least a factor of N, where N is the number of channel realizations used in Section IV. The reader should note, however, that Monte Carlo simulation is still used to evaluate the performance of these methods in Section VI.

A. Instantaneous Equivalent Rate Maximization (IER-Max)

In this method, the principle eigenvector of the respective covariance matrices are chosen as the representatives of the channel. Denote $\mathbf{u}_k$ as the first column of $\mathbf{U}_k$. Define the instantaneous equivalent (IE) rates as

$$\tilde{r}_1(\mathbf{G}) = \log_2\left(1 + \frac{|\mathbf{u}_4^T \mathbf{G} \mathbf{u}_1|^2 \gamma_1}{\|\mathbf{u}_4^T \mathbf{G}\|^2 + 1}\right) \quad \text{and} \tag{20}$$

$$\tilde{r}_2(\mathbf{G}) = \log_2\left(1 + \frac{|\mathbf{u}_3^T \mathbf{G} \mathbf{u}_2|^2 \gamma_2}{\|\mathbf{u}_3^T \mathbf{G}\|^2 + 1}\right) \tag{21}$$

where $\mathbf{G}$ is the relay matrix. In other words, these are the instantaneous rates obtained if the channels are replaced by the dominant eigenvector of their respective covariance matrices. To obtain $\mathbf{G}$, the optimization problem is formulated as (P3):

$$\begin{aligned} \underset{\mathbf{G}}{\text{maximize}} \quad & \tilde{r}(\mathbf{G}) = 0.5(\tilde{r}_1(\mathbf{G}) + \tilde{r}_2(\mathbf{G})) \\ \text{subject to} \quad & \tilde{\gamma}_3 \le \bar{p}_3 \end{aligned} \tag{22}$$

where $\bar{p}_3$ is now considered as an instantaneous power constraint and $\tilde{\gamma}_3$ is the instantaneous transmit power for the equivalent channels $\mathbf{u}_1$ and $\mathbf{u}_2$.

$$\tilde{\gamma}_3 = \|\mathbf{G}\mathbf{u}_1\|^2\gamma_1 + \|\mathbf{G}\mathbf{u}_2\|^2\gamma_2 + \mathrm{Tr}(\mathbf{G}^H\mathbf{G}) \ . \tag{23}$$

Let $\mathbf{F}_A = [\mathbf{u}_1, \mathbf{u}_2] \in \mathbb{C}^{M\times 2}$ and $\mathbf{F}_B = [\mathbf{u}_4^T; \mathbf{u}_3^T] \in \mathbb{C}^{2\times M}$. The following structure of the matrix $\mathbf{G}$ is used.

$$\mathbf{G} = \mathbf{F}_B^H\mathbf{B}\mathbf{F}_A^H \ . \tag{24}$$

A search is carried out over $\mathbf{B} \in \mathbb{C}^{2\times 2}$ to maximize the IE rate $\tilde{r}(\mathbf{G})$. Once the optimization is completed, $\mathbf{G}$ may not satisfy the average power constraint $\bar{p}_3$ in (19). The average transmit power for a relay matrix $\mathbf{G}$ is given by

$$\mathbb{E}[\|\mathbf{G}\mathbf{h}_1\|^2\gamma_1 + \|\mathbf{G}\mathbf{h}_2\|^2\gamma_2 + \mathrm{Tr}(\mathbf{G}^H\mathbf{G})] \tag{25}$$

$$= \mathrm{Tr}[(\gamma_1\mathbf{R}_1 + \gamma_2\mathbf{R}_2 + \mathbf{I}_M)\mathbf{G}^H\mathbf{G}] \ . \tag{26}$$

Therefore, a scaling factor α is used and

$$\mathbf{A} = \alpha\mathbf{G} \tag{27}$$

where $\alpha = \sqrt{\frac{\bar{p}_3}{\mathrm{Tr}[(\gamma_1\mathbf{R}_1+\gamma_2\mathbf{R}_2+\mathbf{I}_M)\mathbf{G}^H\mathbf{G}]}}$.

B. MRR-MRT

This method only requires one step, without optimizing over $\mathbf{B} \in \mathbb{C}^{2\times 2}$. Therefore this method works faster than the previous method. Using the same $\mathbf{F}_A$ and $\mathbf{F}_B$ as in the previous method, the following relay matrix is proposed.

$$\mathbf{A} = \alpha\mathbf{G} = \alpha\mathbf{F}_B^H\mathbf{F}_A^H \tag{28}$$

where, again, α is a normalizing constant for satisfying the average power constraint $\bar{p}_3$.

VI. Simulation Results

In this section, the sum rates obtained by the different schemes are evaluated. For comparison, the curve for the optimal relay matrix given instantaneous CSI, with an instantaneous power constraint $\bar{\gamma}_3$, is displayed as 'OPT.' For all other curves, only covariance CSI is available, and the average power constraint is $\bar{p}_3$.

The channels are modelled as in (11). Consider a uniform linear array (ULA) at the base station, where the antenna spacing is denoted as d. For a AoA/AoD of θ, the steering vector is given by

$$\mathbf{w}_k(\theta) = [1, e^{j2\pi d\sin(\theta)f_k/c}, \cdots, e^{j2\pi(M-1)d\sin(\theta)f_k/c}]^T \tag{29}$$

where f_k is the carrier frequency and c is the speed of light. The following frequencies are chosen. $f_A = f_1 = f_2 =$ 1.8GHz and $f_B = f_3 = f_4 =$ 2.6GHz. d is chosen such that $d \cdot f_A/c = 0.5$. The covariance is calculated as [13]

$$\mathbf{R}_k = \int_{-\pi}^{\pi} \psi_k(\theta)\mathbf{w}_k(\theta)\mathbf{w}_k^H(\theta)d\theta, \tag{30}$$

where $\psi_k(\theta)$ is the ray-density function. The rays arriving at or departing from RL for each of the channels $\mathbf{h}_k$ are assumed to have a uniform density distribution. The nominal AoA/AoD is $\bar{\theta}_k$ and the angular spread is Δ_k. Thus

$$\psi_k(\theta) = \begin{cases} \frac{1}{\Delta_k} & \text{when } \bar{\theta}_k - \Delta_k/2 \le \theta \le \bar{\theta}_k + \Delta_k/2 \\ 0 & \text{otherwise.} \end{cases} \tag{31}$$

In all the simulations, $\Delta_k = \Delta \forall k$, $\bar{\theta}_1 = \bar{\theta}_3$, $\bar{\theta}_2 = \bar{\theta}_4$, and $N = 5000$. Also, $\gamma_1 = \gamma_2 = \bar{\gamma}_3 = \bar{p}_3 = \gamma$ and SNR=$10\log_{10}\gamma$dB. The results are plotted in figures 2 to 5. Unless otherwise stated, the following are the parameters to be assumed in the graphs: $M = 4$, $\Delta = 20^o$, $\bar{\theta}_1 = 10^o$, $\bar{\theta}_2 = 160^o$, and SNR=10dB. To show the estimated sum rate for the optimal design based on covariance feedback, 'SAA 1' denotes $\hat{r}_N(\hat{\mathbf{A}}_N)$ while 'SAA 2' denotes $\hat{c}_N(\hat{\mathbf{A}}_N)$.

A common solution for rate maximization is direct relaying where only a scaled identity matrix is used. The direct method with an average power constraint has the same ergodic rate as that with an instantaneous power constraint.

In Fig. 2, it is seen that with covariance feedback, the low-complexity methods, 'IER-Max' and 'MRR-MRT,' provide a very large gain over the naive scheme of direct relaying. The gain increases as SNR increases. Fig. 3 shows that the proposed low-complexity schemes achieve near-optimal sum rates. For small to moderate angular spreads Δ, the proposed methods have significant rate improvement over direct relaying. However, the value of obtaining covariance CSI compared to instantaneous CSI decreases as Δ increases. The nominal AoA/AoD greatly affects the performance of the direct method, as Fig. 4 shows. The direct method achieves the highest sum rate at $\bar{\theta}_1 = -20^o = \bar{\theta}_2 - 180^o$ because the direct method does not change the direction of the beam. The proposed covariance-based methods are not adversely affected by the nominal angle, due to their ability to steer the beams. In Fig. 5, increasing the number of antennas M increases the sum rate, but at diminishing returns, due to the SIMO/MISO nature of the system. The direct method experiences a decrease in sum rate since it does not take advantage of beamforming, but instead allocates power gain equally.

VII. Conclusion

In this paper, estimation of the optimal rate maximizing relay matrix for the multi-antenna two-way relay given covariance feedback has been proposed. Suboptimal methods with much lower complexity were then proposed. Simulations show that these methods achieve near-optimal performance. These methods also show substantial improvement over direct relaying, especially with more relay antennas, as well as for small to moderate angular spreads at the relay. Also, while the performance of direct relaying is heavily dependent on the nominal angle of arrival, the proposed methods are not adversely affected due to their beamforming capability.

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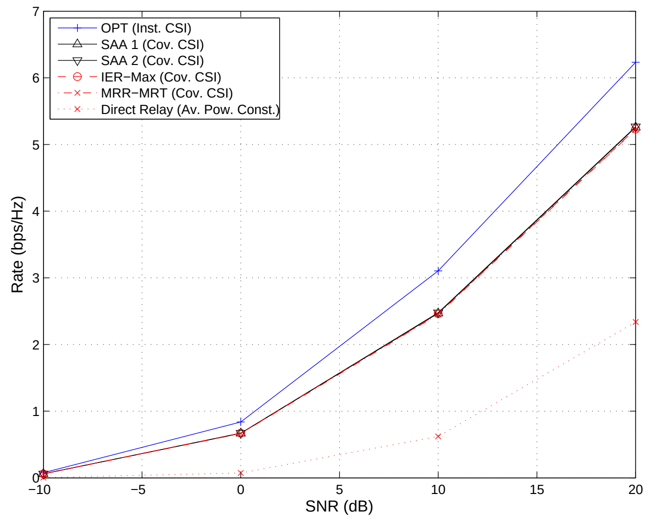

Fig. 2. Comparison of sum rate for different methods.

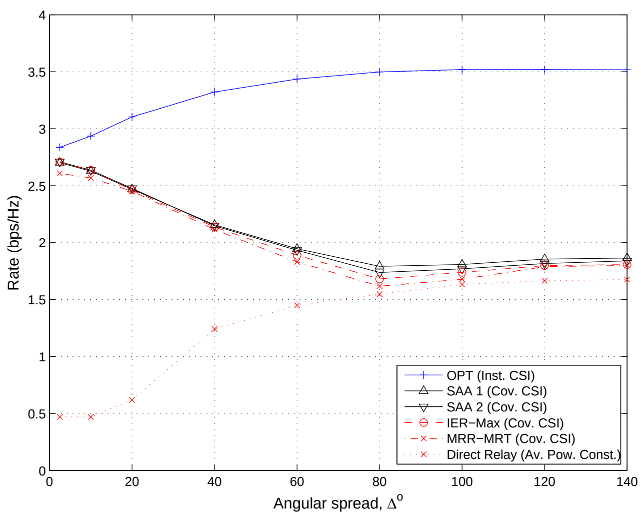

Fig. 3. Effect of angular spread on the sum rate.

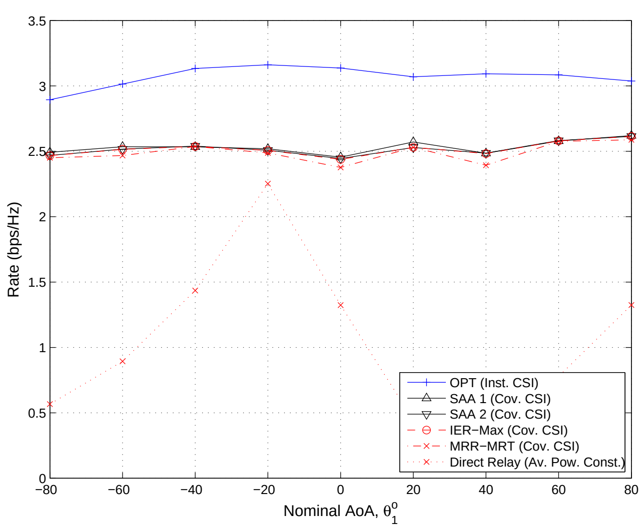

Fig. 4. Graph of sum rate vs the nominal angle $\bar{\theta}_1$ assuming $\bar{\theta}_2 = 160^o$.

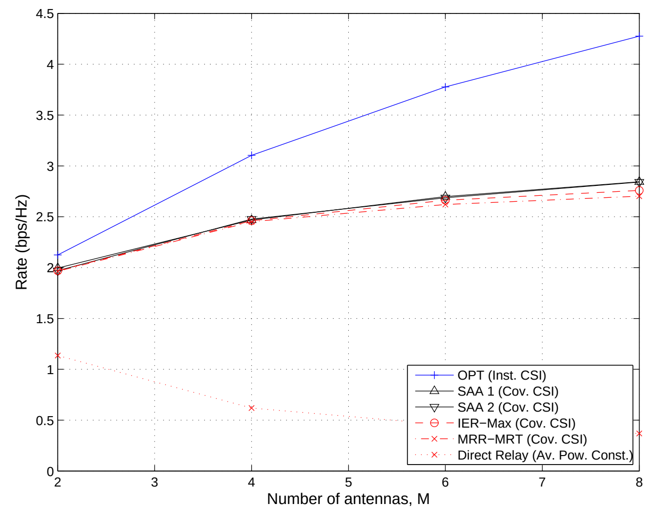

Fig. 5. Performance as the number of relay antennas M increases.

REFERENCES

[1] R. Ahlswede, N. Cai, S.-Y. R. Li, and R. W. Yeung, “Network Information Flow,” *IEEE Trans. Inform. Theory*, vol. 46, no. 4, pp. 1204–1216, Jul. 2000.

[2] S. Zhang, S.-C. Liew, P. P. Lam “Physical-Layer Network Coding,” *Proc. of MobiCom*, pp. 358–365, 2006.

[3] S. Katti, S. Gollakota, and D. Katabi, “Embracing Wireless Interference: Analog Network Coding,” *Proc. of ACM SIGCOMM*, pp. 397–408, 2007.

[4] P. Popovski and H. Yomo, “Physical Network Coding in Two-Way Wireless Relay Channels,” *Proc. Int. Conf. Commun.*, pp. 707–712, Jun. 2007.

[5] Y.-C. Liang and R. Zhang, “Optimal Analogue Relaying with Multi-Antennas for Physical Layer Network Coding,” *Proc. Int. Conf. Commun.*, accepted for publication, May. 2008.

[6] T. J. Oechtering, C. Schnurr, I. Bjelakovic, and H. Boche, “Broadcast Capacity Region of Two-Phase Bidirectional Relaying,” *IEEE Trans. Inform. Theory*, vol. 54, no. 1, pp. 454–458, Jan. 2008.

[7] B. Rankov and A. Wittneben, “Spectral Efficient Protocols for Half-Duplex Fading Relay Channels,” *IEEE J. Select. Areas Commun.*, vol. 25, no. 2, pp. 379–389, Feb. 2007.

[8] E. Visotsky and U. Madhow, “Space-time transmit precoding with imperfect feedback,” *IEEE Trans. Inform. Theory*, vol. 47, no. 6, pp. 2632–2639, Sep. 2001.

[9] M. Vu and A. Paulraj, “Capacity optimization for Rician correlated MIMO wireless channels,” *Proc. Asilomar Conf. Signals, Systems, Computers*, pp. 133–138, Oct. 2005.

[10] A. M. Tulino, A. Lozano, and S. Verdu, “Capacity-Achieving Input Covariance for Single-User Multi-Antenna Channels,” *IEEE Trans. Wireless Commun.*, vol. 5, no. 3, pp. 662–671, Mar. 2006.

[11] A. Soysal and S. Ulukus, “Optimum Power Allocation for Single-User MIMO and Multi-User MIMO-MAC with Partial CSI,” *IEEE J. Select. Areas Commun.*, vol. 25, no. 7, pp. 1402–1412, Sep. 2007.

[12] M. Herdin, “MIMO Amplify-and-Forward Relaying in Correlated MIMO Channels,” *Proc. IEEE ICICS*, pp. 796–800, Dec. 2005.

[13] Y.-C. Liang and F. P. S. Chin, “Downlink channel covariance matrix (DCCM) estimation and its applications in wireless DS-CDMA systems,” *IEEE J. Select. Areas Commun.*, vol. 19, no. 2, pp. 222–232, Feb. 2001.

[14] A. B. Gershman and N. D. Sidiropoulos, *Space-Time Processing for MIMO Communications*, Wiley, 2005.

[15] A. J. Kleywegt, A. Shapiro, and T. Homem-de-Mello, “The sample average approximation method for stochastic discrete optimization,” *SIAM Journal on Optimization*, vol. 12, no. 2, pp. 479–502, 2002.