Efficient Resource Allocation for Power Minimization in MIMO-OFDM Downlink

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***Abstract*—In a downlink system using multiple-input multiple-output orthogonal frequency division multiplexing (MIMO-OFDM), the subcarrier and power allocations can be optimized to minimize the overall transmit power given user target rates. If done efficiently, this resource allocation helps to reduce the interference ingress to neighbouring cells and limits the power consumption at the base station. The optimal solution can be found with a complexity of $\mathcal{O}(K^M)$ for a system with K users and M subcarriers. This paper proposes an efficient method using a dual decomposition that has a lower complexity of only $\mathcal{O}(MK)$. Linear beamforming is assumed at both the transmitter and the receiver ends. Frequency-flat fading may adversely affect OFDM resource allocation if using a dual decomposition based approach. Flat fading management is thus proposed by using a certain *dual proportional fairness*, that handles all fading scenarios, including flat or partially frequency-selective fading. Simulations show fast convergence of the algorithm, quickly approaching the optimal solution.**

I. Introduction

In cellular systems, proper resource allocation for multiple-input multiple-output orthogonal frequency division multiplexing (MIMO-OFDM) [1] allows more users to be supported at any given time and provides higher data rates per user. Many communication problems can be solved by efficient convex optimization techniques [2, 3]. For example, [4–6] solve the flat fading MIMO downlink power minimization, with the help of the uplink-downlink duality [7–10]. Dirty paper coding (DPC) is assumed at the base station (BS) during the downlink. Time-sharing between the different decoding orders is required when the target rate-tuple lies on the convex hull of the respective vertices in the capacity region. The time-sharing scheme can be found by a linear program [4].

If directly applying the above nonlinear methods to the MIMO-OFDM downlink, each subcarrier requires a different encoding order. While these solutions are optimal in minimizing the sum transmit power, the drawback is that hardware complexity is raised. As an alternative, this paper considers linear processing. For a flat fading MIMO broadcast channel, zero-forcing (ZF) beamforming with time division multiple access (TDMA) has been shown to achieve a sum rate close to the optimal DPC scheme when the number of users is large [14].

[12] obtains subcarrier and bit allocations with a goal of minimizing the overall transmit power while maintaining a target BER for a multiuser MIMO-OFDM system. For each subcarrier, the user that achieves the maximum SNR is selected. For the SISO case, an efficient OFDM downlink resource allocation has been developed in [13], which also does not have the complexity of different encoding orders, since there is only one user per subcarrier. Thanks to a dual decomposition approach, user selection for a subcarrier is only decided by a metric dependent on that subcarrier alone, vastly reducing the complexity. [12] and [13] are excellent for frequency-selective fading OFDM channels. Unfortunately, frequency-flat OFDM channels, if they occur, would result in an inability to guarantee user rates because the decision to select a particular user for one subcarrier would be repeated for all the subcarriers.

In this paper, an efficient method is designed to minimize the total transmit power for the MIMO-OFDM downlink, subject to individual user rate constraints, requiring only linear transmit and receive processing. By considering the Lagrangian of the sum power objective function and applying a dual decomposition, the problem is broken down into M individual subproblems, where M is the number of subcarriers. The complexity is thus reduced from one exponential in M to one linear in M. Given that M is typically large for multi-carrier systems, this represents a huge complexity reduction. The supergradient of the dual function is used to update the Lagrange multipliers. As mentioned earlier, methods based on dual decomposition could possibly suffer from a uniformity among the subcarriers, resulting in large oscillations within the algorithm. A solution based on a *dual proportional fairness* is proposed to handle the event of frequency-flat fading as well.

Section II describes the channel model. The optimal solution to resource allocation for power minimization is given in Section III. An efficient solution based on dual decomposition is developed in Section IV. To handle the event of flat fading channels, a modification based on a dual proportional fairness is introduced in Section V. Simulation results are given in Section VI. Finally, conclusions are drawn in Section VII.

Notations:

Vectors and matrices are denoted by boldface letters. $(\cdot)^T$ and $(\cdot)^H$ denote the transpose and conjugate transpose operations respectively. $\mathbb{E}[\cdot]$ stands for to the expectation operator. $\mathbf{I}_N$ denotes the $N \times N$ identity matrix. $\mathbf{A} = \mathrm{blkd}(\mathbf{A}_1, \mathbf{A}_2, \ldots, \mathbf{A}_K)$ represents the block diagonal matrix consisting of $\mathbf{A}_k$ as its diagonal entries.

II. Channel Model

In this section, the downlink channel model is given. Consider a cellular-based MIMO-OFDM system with a BS communicating with K user terminals via M subcarriers. Suppose the BS is equipped with N_T antennas and the k-th user terminal has n_k antennas. Denote $N_R = \sum_{k=1}^{K} n_k$ as the total number of receive antennas. Let $\sigma_{k,m}$ indicate the presence of the k-th user on subcarrier m, where $\sigma_{k,m} = 1$ if present and 0 if not. It is assumed that only 1 user is selected on each subcarrier. Let the rank of the channel matrix of user k on subcarrier m be denoted by $\eta_{k,m}$, where $0 \leq \eta_{k,m} \leq \min(n_k, N_T), \forall m$. The baseband input-output relationship is represented as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \ , \tag{1}$$

where $\mathbf{x} = [\mathbf{x}_1^T, \ldots, \mathbf{x}_M^T]^T$ is the transmit signal vector, $\mathbf{H} = \mathrm{blkd}(\mathbf{H}_1, \ldots, \mathbf{H}_M)$ is the channel, $\mathbf{y} = [\mathbf{y}_1^T, \ldots, \mathbf{y}_M^T]^T$ is the receive signal vector, and $\mathbf{n}$ is the $MN_R \times 1$ noise vector. Assume that the noise is zero-mean, circularly symmetric complex Gaussian (CSCG) with $\mathbb{E}[\mathbf{n}\mathbf{n}^H] = N_0\mathbf{I}$, and $\mathbf{n}$ is independent of $\mathbf{x}$. For the m-th subcarrier, (1) can be interpreted as

$$\mathbf{y}_m = \mathbf{H}_m\mathbf{x}_m + \mathbf{n}_m \ , \tag{2}$$

where $\mathbf{H}_m = [\mathbf{H}_{1,m}^T, \ldots, \mathbf{H}_{K,m}^T]^T$ is the $N_R \times N_T$ random MIMO channel and $\mathbf{y}_m = [\mathbf{y}_{1,m}^T, \ldots, \mathbf{y}_{K,m}^T]^T$ is the $N_R \times 1$ receive signal vector on subcarrier m.

III. Optimal Solution for Power Minimization

In this section, the problem of power minimization is formulated mathematically and the optimal solution is derived. The objective is to find the optimal subcarrier allocation $\{\sigma_{k,m}\}$ and power allocation $\{p_{k,m}\}$ that minimize the overall transmit power subject to satisfying each user's normalized data rate requirement $\bar{R}_k$ bits per sec per Hz (bps/Hz). (For M subcarriers, each with bandwidth ω, the overall rate for user k is $M\bar{R}_k\omega$ bps. $M\bar{R}_k$ bits are transmitted for user k in the duration of one OFDM symbol i.e. one channel use.) Define the singular value decomposition of user k's channel on subcarrier m as

$$\mathbf{H}_{k,m} = \mathbf{U}_{k,m}\mathbf{S}_{k,m}\mathbf{V}_{k,m}^H \ . \tag{3}$$

Let $r_{k,m}$ be the rate of user k on subcarrier m, which can be written as

$$r_{k,m} = \sum_{l=1}^{\eta_{k,m}} \log_2\left(1 + \frac{\tilde{p}_{k,m,l}s_{k,m,l}^2}{\Gamma N_0}\right) \ , \tag{4}$$

where $s_{k,m,l}$ is the l-th diagonal element of $\mathbf{S}_{k,m}$, $\tilde{p}_{k,m,l}$ is the power loading on this subchannel, $p_{k,m} = \sum_{l=1}^{\eta_{k,m}} \tilde{p}_{k,m,l}$, and Γ is the SNR gap. Mathematically, the optimization can be expressed as

$$\begin{aligned} \min_{\{\sigma_{k,m}\},\{p_{k,m}\}} \quad & \sum_{m=1}^{M}\sum_{k=1}^{K} p_{k,m} \\ \text{subject to} \quad & \sum_{m=1}^{M} r_{k,m} \geq M\bar{R}_k \ , \quad \forall k \\ & p_{k,m} \geq 0 \ , \quad \forall k, m \ . \end{aligned} \tag{5}$$

If the subcarrier assignment $\{\sigma_{k,m}\}$ is fixed, the power allocation can be found for each user separately. If user k is of interest, the problem becomes

$$\begin{aligned} \min_{\{p_{k,m}\}} \quad & \sum_{m=1}^{M} p_{k,m} \\ \text{subject to} \quad & \sum_{m=1}^{M} r_{k,m} \geq M\bar{R}_k \\ & p_{k,m} \geq 0 \ , \quad \forall m \\ & p_{k,m} = 0 \ , \quad \sigma_{k,m} = 0 \ . \end{aligned} \tag{6}$$

Water-filling [18] can be then carried out over user k's eigen-channels across all the subcarriers to find the optimal power and rate allocation. In order to obtain the globally optimal solution, an exhaustive search is needed over all the subcarrier assignments $\{\sigma_{k,m}\}$ to find the minimum transmit sum power. Thus, K water-filling procedures over Mn_k singular values have to be carried out for each of K^M possibilities.

IV. Efficient Solution for Power Minimization

While the solution described in the previous section is optimal, the complexity is large because of an exhaustive search over a large set of possible subcarrier allocations. In this section, an efficient solution to the power minimization problem is derived based on a dual decomposition.

The Lagrangian of the optimization problem (5) is

$$\mathcal{L}_1 = \sum_{m=1}^{M}\sum_{k=1}^{K} p_{k,m} + \sum_{k=1}^{K}\mu_k\left(M\bar{R}_k - \sum_{m=1}^{M} r_{k,m}\right) \ , \tag{7}$$

where μ_k are the Lagrange multipliers and $r_{k,m}$ is given by (4). If the μ_k are fixed, users can be selected on a per subcarrier basis. (7) can be written as

$$\mathcal{L}_1 = \sum_{m=1}^{M}\mathcal{L}_2(m) + \sum_{k=1}^{K}\mu_k M\bar{R}_k \ , \tag{8}$$

where

$$\mathcal{L}_2(m) = \sum_{k=1}^{K}\left(p_{k,m} - \mu_k r_{k,m}\right) \ . \tag{9}$$

Consequently, the problem is decomposed into M independent subproblems. Assume that the user selection $\{\sigma_{k,m}\}$ has been fixed. Considering one subcarrier,

$$\mathcal{L}_2(m) = \sum_{k=1}^{K}\sum_{l=1}^{\eta_k}\left(\tilde{p}_{k,m,l} - \mu_k\log_2\left(1 + \frac{\tilde{p}_{k,m,l}s_{k,m,l}^2}{\Gamma N_0}\right)\right) \ . \tag{10}$$

$\mathcal{L}_2(m)$ can then be minimized for each user separately in order to calculate $\tilde{p}_{k,m,l}$. By applying the water-filling procedure, the power allocation and rate for the l-th subchannel of user k can be found:

$$\tilde{p}_{k,m,l} = \max\left\{\frac{\mu_k}{\ln 2} - \frac{\Gamma N_0}{s_{k,m,l}^2}\ ,\ 0\right\}\ , \tag{11}$$

$$\tilde{r}_{k,m,l} = \log_2\left(\max\left\{\frac{\mu_k s_{k,m,l}^2}{\ln 2\ \Gamma N_0}\ ,\ 1\right\}\right)\ . \tag{12}$$

Consequently, a search over K possible users on subcarrier m can be carried out to select the best user that minimizes $\mathcal{L}_2(m)$. Once this is carried out on all the subcarriers, the minimum value of $\mathcal{L}_1$ obtained is called the dual function. The algorithm starts by calculating the dual function value for an initial set of Lagrange multipliers μ_k. The Lagrange multipliers are then updated iteratively to maximize the dual function value. The maximum dual function value is thus called the dual solution while the solution to the original optimization (5) is called the primal solution. The dual solution obtained is always a lower bound to the primal solution and the difference between these two is known as the duality gap.

Overall, for M subcarriers, the complexity of the search is $\mathcal{O}(MK)$. This is less than the $\mathcal{O}(K^M)$ complexity in the optimal solution of Section III. In multicarrier systems where the number of subcarriers M is typically large, this represents a huge reduction in complexity. Furthermore, when M is large, the duality gap is negligible [15]. For a certain channel realization, if the duality gap happens to be zero, the efficient solution offered in this section coincides exactly with the optimal solution. On the other hand, if the duality gap is not zero, this efficient solution is near-optimal in terms of sum power minimization for target rates.

Next, the update of the Lagrange multipliers is described. Define $\mathbf{d} = [d_1, \ldots, d_K]^T$ as a scaled version of the supergradient [16] of the dual function at the current set of Lagrange multipliers, where

$$d_k = \bar{R}_k - \frac{1}{M}\sum_{m=1}^{M} r_{k,m}\ . \tag{13}$$

Starting from an initial value, the Lagrange multipliers are updated in the positive supergradient direction in order to maximize the dual function.

$$\mu_k(\tau+1) = \max\left\{\mu_k(\tau) + \delta\ d_k\ ,\ 0\right\}\ , \tag{14}$$

where τ represents the iteration number and δ is a small step size. μ_k can be interpreted as the reward for user k to increase its rate. The direction of (14) suggests that if the rate of user k falls below its target rate, its rate reward μ_k should be increased. On the other hand, if user k exceeds its rate requirement, μ_k should be decreased but the rate reward should not fall below zero. During the optimization process, the dual rates for the users,

$$r_k = \sum_{m=1}^{M} r_{k,m}\ , \tag{15}$$

gradually approach the rate requirements $M\bar{R}_k$. However, at any point in time, the current subcarrier selections $\{\sigma_{k,m}\}$ can be captured to solve for the optimal minimum power solution given target rates. As the optimization proceeds, this power value for guaranteed rates will tend to decrease and approach the dual function value. Unlike algorithms such as steepest-descent, the dual function value is not guaranteed to increase monotonically with each iteration. Therefore, the algorithm keeps track of the the subcarrier selection $\{\sigma_{k,m}\}$ that provides the minimum sum power over all the previous iterations.

V. Dual Proportional Fairness

The efficient algorithm proposed in Section IV is immediately applicable to harsh channel conditions. Due to the frequency-selective nature of the channel, the user selection for each subcarrier is optimized to minimize the overall transmit power. However, frequency-flat fading channels may pose a problem. For example, for the case of perfectly flat fading, the same user would be selected for every subcarrier. Thus, only one user is allocated all the subcarriers at any one time. This causes serious oscillation problems for the algorithm. The subcarrier allocation $\{\sigma_{k,m}\}$ provided by the optimization is unable to guarantee all the users' target rates.

In this section, a concept called *dual proportional fairness* is proposed, drawing its inspiration from the principle of proportional fairness [17] where randomness is exploited. With this concept, flat fading management can be carried out easily to handle the possibility of frequency-flat fading. In proportional fairness, the nature of the fluctuating channel helps the design of the time schedules; in dual proportional fairness, the nature of the fluctuating dual rates helps the design of the subcarrier allocation.

A. Principle of Dual Proportional Fairness

In the dual optimization method proposed in Section IV, the Lagrange multipliers define a tangent plane in a graph of power versus user rates. In this graph, there are several power surfaces, each representing a different subcarrier allocation. For any given tuple of user target rates, the pointwise minimum of all these power surfaces is the minimum total transmit power achievable. As the number of subcarriers M increases, a larger number of power surfaces corresponding to various subcarrier allocations are generated. The pointwise minimum of these power surfaces therefore tends to assume a convex shape. While the optimization is in progress, the tangent plane remains in contact with this minimum surface. The current dual rates for all the users are given by the coordinates at the point of contact. The purpose of the Lagrange multiplier update is to shift this tangent plane such that the dual rates approach the users' rate requirements. The minimum sum power is obtained when the dual rates reach the target rates.

Flat fading channels give rise to oscillation problems because a tangent plane can touch several power surfaces at collinear points. Suppose there are 2 users. In simulations, it is impossible for the tangent plane to touch a middle power surface, corresponding to a certain fraction of total subcarriers allocated to user 1 and the remaining fraction to user 2, without touching the other power surfaces. Consequently, the algorithm oscillates between giving all the subcarriers to user 1 or all to user 2. As a result, each user's rate varies dramatically between zero and a value exceeding its target rate.

The concept of *dual proportional fairness* is now described. In frequency-flat fading, the power allocations and rates for user k on all the subcarriers are identical:

$$p_{k,m} = \hat{p}_k \ , \quad \forall m \tag{16}$$

$$r_{k,m} = \hat{r}_k \ , \quad \forall m \tag{17}$$

$$p_k = M_k \hat{p}_k \tag{18}$$

$$r_k = M_k \hat{r}_k \ , \tag{19}$$

where M_k is the number of subcarriers allocated to user k and $\sum_{k=1}^{K} M_k = M$. Consider the case of two users. The possible coordinates given by the optimization algorithm are

$$(M\hat{r}_1 \ , \ 0 \ , \ M\hat{p}_1) \tag{20}$$

$$(0 \ , \ M\hat{r}_2 \ , \ M\hat{p}_2) \ . \tag{21}$$

Another coordinate, not given by the original optimization, is also possible:

$$(M_1\hat{r}_1 \ , \ M_2\hat{r}_2 \ , \ M_1\hat{p}_1 + M_2\hat{p}_2) \ . \tag{22}$$

Clearly, these three coordinates are collinear. This concept can be extended to more than 2 users. The task that remains is to obtain the right combination of $\{M_k\}$ that minimizes the transmit power. The following three steps are proposed.

1. Identify the flat fading users.
2. Identify the flat fading groups.
3. Distribute the subcarriers proportionally for each fading group.

B. Algorithm for Flat Fading Management

1) Identify the flat fading users: Flat fading users are identified as users with rates that oscillate largely or drop to zero in the current and previous 9 iterations. Assume there are K_{ff} such users.

2) Identify the flat fading groups: For each flat fading user, check back to see when it had received a dual rate higher than its target rate. (If he had not, the flat fading management cannot be done at the moment.) Denote $\bar{M}_k$ as the minimum number of subcarriers user k needs to just fulfill his rate requirement. Next, consider all users pairwise. Take user 1 and user 2 for example. Find out where the subcarriers allocated to user 1, $\mathbf{\Sigma}_1$, overlaps with the subcarriers of user 2, $\mathbf{\Sigma}_2$. If they do overlap, users 1 and 2 are in the same group G_v. The union of subcarriers is taken as the flat fading subcarriers of this group, $\mathbf{\Sigma}_{G_v}$. Continue this process for all K_{ff} flat fading users. Users that are not interlinked in this manner are placed in separate flat fading groups. Assume there are K_v users in each fading group G_v.

3) Distribute the subcarriers proportionally for each fading group: Let there be $\tilde{M}_v$ flat fading subcarriers in G_v. First assume the special case of flat fading over all the subcarriers. Users are allocated subcarriers cyclically until user k gets a maximum of

$$\text{round}\left[\frac{\bar{M}_k}{\sum_{k\in G_v} \bar{M}_k}\tilde{M}_v\right] \tag{23}$$

subcarriers. To make sure all the subcarriers get allocated, the last user gets all the remaining subcarriers.

An additional modification to (23) allows the algorithm to handle the general case of partially frequency-selective channels. Consider for example the case of two users. In the graph of power versus user rates, only a subset of subcarrier allocations result in collinear points of contact with the tangent plane. This time, oscillations do occur but they are not between zero and very high rates. Instead, each user's dual rate oscillates above and below its rate requirement while its dual rate does not drop to zero. Practically, taking the current subcarrier allocation $\{\sigma_{k,m}\}$ still allows the user rates to be guaranteed, but this is at an expense of higher transmit power that also oscillates largely.

In the following, a modification to (23) is developed that allows smooth convergence for the general case of partially frequency-selective channels. For each user, find the subcarriers that were allocated to this user for every of the current and previous 9 iterations. Let there be $M_{k,\min}$ such subcarriers. Let $\tilde{\mathbf{\Sigma}}_{G_v}$ be the subcarriers of group G_v with the subcarriers corresponding to $M_{k,\min}$ of all flat fading users removed. Let there be $\tilde{\tilde{M}}_v$ flat fading subcarriers in $\tilde{\mathbf{\Sigma}}_{G_v}$. These subcarriers are distributed in a similar manner as in the previous section. All flat fading users get allocated their respective $M_{k,\min}$ subcarriers. The initial estimated number of subcarriers each flat fading user would get from $\tilde{\mathbf{\Sigma}}_{G_v}$ is

$$\bar{\bar{M}}_k = \max\left\{\bar{M}_k - M_{k,\min} \ , \ 0\right\} \ . \tag{24}$$

Users are allocated subcarriers cyclically until user k gets a maximum of

$$\text{round}\left[\frac{\bar{\bar{M}}_k}{\sum_{k\in G_v} \bar{\bar{M}}_k}\tilde{\tilde{M}}_v\right] \tag{25}$$

subcarriers. Again, to handle any rounding errors, the last user is allocated all the remaining subcarriers. Subcarriers that are not affected by the flat fading management are assigned the same subcarriers as given by the original solution without any flat fading management.

VI. Simulation Results

This section evaluates the performance of the proposed algorithm. A MIMO-OFDM downlink with $M = 32$ subcarriers is investigated. The MIMO setup is $4\times[4, 4, 4]$, where there are 3 users and there are 4 antennas on the base station and each of the user terminals. The rate requirement is set at 5 bps/Hz for each user while the SNR gap $\Gamma = 3$ dB. Adaptation of the step size δ for fast convergence is designed in [11] and is used here. The algorithm in section V is used in all the simulations. The SNR achieved is defined as $\frac{\sum_{m=1}^{M}\sum_{k=1}^{K} p_{k,m}}{MN_0}$. Therefore the dual function is also scaled by $\frac{1}{MN_0}$ for comparison. Figures

1 and 2 show the convergence behaviour of the algorithm. The vertical lines at the first few iterations indicate that the user target rates are not feasible because at least one user did not get any subcarriers. The '$\times$' denotes the sum power for guaranteed target rates. Fig. 1 shows the case of a frequency-selective channel with 9 taps. The power given by the efficient algorithm approaches the dual function value closely, suggesting that the duality gap is almost zero. The flat fading scenario is shown in Fig. 2. Despite the widely oscillating dual rates, the algorithm quickly achieves a sum power that guarantees all user rate targets, thanks to the dual proportional fairness principle.

VII. Conclusion

Optimal resource allocation for power minimization in the MIMO-OFDM downlink subject to user target rates has been considered. This paper proposes an efficient algorithm that obtains the subcarrier, power, and rate allocations through the use of a dual decomposition, thereby achieving a much lower complexity. In typical multicarrier systems where the duality gap is small, this solution is close to the optimal solution. A concept called *dual proportional fairness* is proposed to realize good performance in all fading scenarios, even in frequency-flat fading. Simulations show fast convergence of the algorithm to a near-optimal power.

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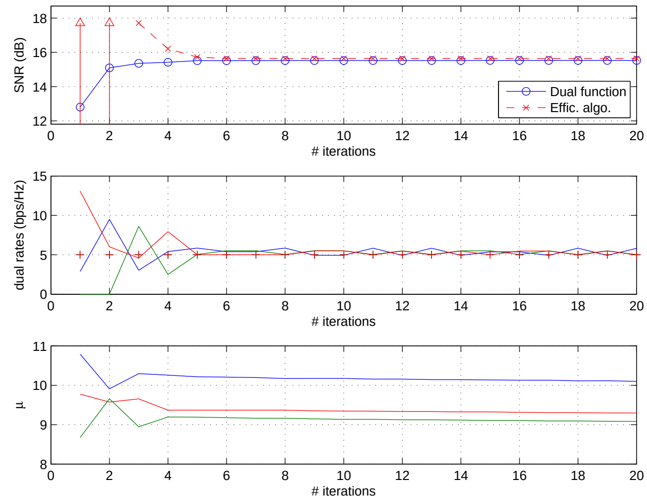

Fig. 1. Typical convergence behaviour for frequency-selective channel.

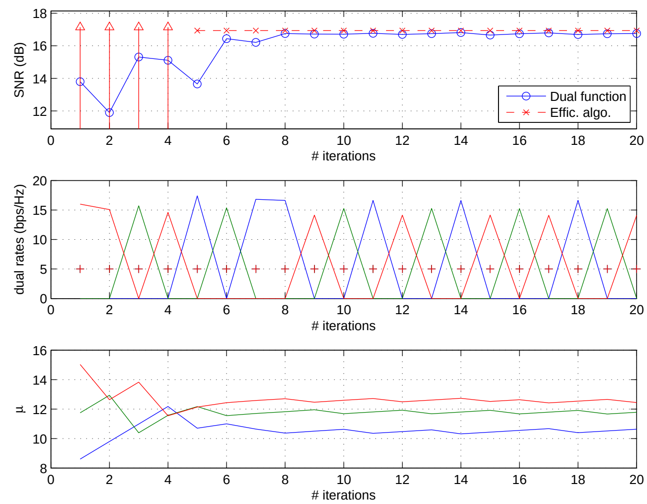

Fig. 2. Typical convergence behaviour for flat fading channel.

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