User Ordering and Subchannel Selection for Power Minimization in MIMO Broadcast Channels using BD-GMD

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***Abstract*—This paper studies the power minimization problem for the MIMO broadcast channel. The optimal solution involves interference-balancing (IB) and iterative convex optimization procedures. In this paper, the zero-forcing (ZF) problem is considered, with dirty paper coding (DPC), resulting in a simple non-iterative implementation using the block diagonal geometric mean decomposition (BD-GMD). Subchannel selection is applied in order to approach the performance of the optimal IB solution. Optimal and near-optimal solutions are provided to find the encoding order and subchannel selection for each user. The advantages of the methods proposed are their non-iterative nature and much reduced computational complexity. Simulations run on both uncorrelated and correlated channels show that a transmit power close to the optimal IB solution can be reached.**

I. Introduction

In multiple-input multiple-output (MIMO) wireless communications, multiple antennas are deployed at both the transmitter and the receiver, giving much capacity improvement without using additional power or bandwidth. For a MIMO broadcast channel (BC), with channel state information at the transmitter (CSIT), the block diagonal geometric mean decomposition (BD-GMD) [8, 9] can be applied with dirty paper coding (DPC) to create identical SNRs for every data stream of each particular user. Equal-rate modulation can then be used on these data streams. The benefits of using the equal-rate modulation include the excellent BER, for uncoded systems [1], in addition to the reduced transceiver complexity. The BD-GMD is a multiuser extension of the geometric mean decomposition (GMD) [1] for point-to-point communications. Equal-rate modulation is also useful where there is a restriction on constellation size, especially in practical wireless links. For example, very higher order modulation may be required for good subchannels when using a communication strategy based on singular value decomposition (SVD). This may be impractical due to the presence of phase noise and synchronization errors.

In multiuser systems, users may be placed at different distances from the base station (BS), resulting in different channel strengths. Furthermore, user rate requirements may be different. While satisfying the rate requirements, it is important to minimize the transmit power of the BS to reduce the interference it causes to other BSs. Convex optimization [2, 3] offers iterative methods to solve several non-linear communications problems. Using the uplink-downlink duality [4] as well as convex optimization techniques, [5–7] are significant papers that address the power minimization problem, for the case of users with multiple antennas, assuming perfect CSIT.

For the papers mentioned above, convex optimization provides the optimal user orderings and minimum power. This optimal solution is referred to as interference-balancing (IB), as opposed to zero-forcing (ZF), since noise is taken into account, and interference is allowed between the subchannels. The complexity is high due to its inherent iterative nature, heavy computational load in each iteration, and large number of iterations. Simple ZF-based solutions have to be found that approach the optimal. Although suboptimal, these solutions help in reducing the complexity of the hardware. In [10], non-iterative precoding methods were designed for power minimization given user rate requirements. Fast computation of the optimal (ZF case) user ordering is done with less than $KK!$ determinant calculations for a system with K users. Using a method called successive closest match (SCM), an ordering that is close to the optimal is found with only $K(K+1)/2$ determinant calculations. To do so, it was assumed that all available subchannels are used, meaning that the number of subchannels used for each user is equal to the number of its antennas.

For MIMO channels, a common phenomenon encountered is channel correlation. As the MIMO channels become rank deficient, it would be better to use only a selection of the available subchannels. For point-to-point communication using GMD, allowing subchannel selection may result in a lower transmit power for the same target rate. Correspondingly, for the MIMO broadcast channel, this paper proposes BD-GMD with subchannel selection (BD-GMD-SS). In the multiuser scenario, there is an extra advantage of subchannel selection for a user because it frees up more spatial degrees of freedom for the later encoded users. This paper describes techniques to find the best user ordering and subchannel selection for the BD-GMD-SS. Simulations show that the minimum power solution using BD-GMD-SS can be be found over a hundred times faster than the optimal IB solution, with a sum power close to that. To reduce the complexity even further, a suboptimal ordering method is proposed with little performance loss.

This paper is organized as follows. Section II gives the channel model. Following that, Section III describes a single-user GMD with subchannel selection. Next, the power minimization for a given user ordering and subchannel selection for the MIMO broadcast channel is derived in Section IV. The complexity of finding the best user ordering and subchannel selection is discussed in Section V. Then in Section VI, an efficient method to find the user ordering and subchannel selection is proposed. Simulation results are provided in Section VII and the conclusion is contained in Section VIII.

Notations:

Let $\mathbf{I}_N$ denote the $N \times N$ identity matrix. $\mathrm{diag}(\mathbf{L})$ is the diagonal matrix with elements from the main diagonal of $\mathbf{L}$. Let $\mathbf{A} = \mathrm{blkd}(\mathbf{A}_1, \mathbf{A}_2, \ldots, \mathbf{A}_K)$ represent the block diagonal matrix with $\mathbf{A}_k$ as diagonal blocks. $\mathbb{E}[\cdot]$, $(\cdot)^T$, and $(\cdot)^H$ represent the expectation, transpose, and conjugate transpose respectively. $\mathbb{C}^{M\times N}$ is the set of complex $M \times N$ matrices.

II. Channel Model

Given a cellular system with one BS and K mobile users, consider the *broadcast channel* from the BS to the mobile users. The BS is equipped with N_T antennas, and the k-th mobile user has n_k antennas. Let $N_R = \sum_{i=1}^{K} n_i$ be the total number of receive antennas, where $N_T \geq N_R$. The input-output relation can be represented as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{u} \ , \tag{1}$$

where $\mathbf{x} \in \mathbb{C}^{N_T\times 1}$ is the transmit signal vector at the BS, $\mathbf{y} \in \mathbb{C}^{N_R\times 1}$ is the receive signal vector with $\mathbf{y} = [\mathbf{y}_1^T, \cdots, \mathbf{y}_K^T]^T$, and each $\mathbf{y}_k \in \mathbb{C}^{n_k\times 1}$ is the receive signal vector of user k. $\mathbf{H} = [\mathbf{H}_1^T, \ldots, \mathbf{H}_K^T]^T$, where each $\mathbf{H}_k \in \mathbb{C}^{n_k\times N_T}$ is the channel of user k. Assume that the noise vector $\mathbf{u}$ has independent and identically distributed (i.i.d.) zero-mean circularly symmetric complex Gaussian (CSCG) elements with $\mathbb{E}[\mathbf{u}\mathbf{u}^H] = N_0\mathbf{I}$, and $\mathbf{u}$ is independent of $\mathbf{x}$. Assume also that $\mathbb{E}[\|\mathbf{x}\|^2] = E_s$. Denote this downlink model by $N_T \times [n_1, \ldots, n_K]$.

III. Single-user GMD with Subchannel Selection

For a matrix $\mathbf{H}$, the single-user GMD is [1] $\mathbf{H} = \mathbf{P}\mathbf{L}\mathbf{Q}^H$. Here, $\mathbf{P}$ and $\mathbf{Q}$ are square unitary matrices, and $\mathbf{L}$ is a lower triangular matrix with all the diagonal elements equal, and given by the geometric mean of the singular values of $\mathbf{H}$, $\bar{s} = \prod_{i=1}^{N} s_i$, where N is the size of the smallest dimension of $\mathbf{H}$. Subchannel selection may provide a lower transmit power for a given target rate.

To do this, SVD is performed on the channel matrix. Next, GMD is applied to the first η singular values. For example, suppose that only the first 2 out of 3 singular values are used for a 3×3 channel matrix $\mathbf{H}$.

$$\mathbf{H} = \mathbf{U} \begin{bmatrix} \mathbf{S}' & \\ & s_3 \end{bmatrix} \mathbf{V}^H \tag{2}$$

$$= \begin{bmatrix} \mathbf{U}' & \mathbf{u}_3 \end{bmatrix} \begin{bmatrix} \mathbf{P}'\mathbf{L}\mathbf{Q}'^H & \\ & s_3 \end{bmatrix} \begin{bmatrix} \mathbf{V}'^H \\ \mathbf{v}_3^H \end{bmatrix} \tag{3}$$

$$= \begin{bmatrix} \mathbf{P} & \mathbf{u}_3 \end{bmatrix} \begin{bmatrix} \mathbf{L} & \\ & s_3 \end{bmatrix} \begin{bmatrix} \mathbf{Q}^H \\ \mathbf{v}_3^H \end{bmatrix} \ , \tag{4}$$

where the sizes of the respective matrices should be clear. Note that the first and last matrices of (4) are unitary. After pre- and post-multiplying (4) by $\mathbf{P}^H$ and $\mathbf{Q}$ respectively,

$$\mathbf{P}^H\mathbf{H}\mathbf{Q} = \mathbf{L} \ , \tag{5}$$

where $\mathbf{L}$ is a lower triangular matrix with diagonal elements all equal to the geometric mean of the first η singular values of $\mathbf{H}$. Let this equation (5) be called GMD-SS, where 'SS' refers to 'subchannel selection.'

IV. Power Minimization for a Given User Ordering and Subchannel Selection

In [10], a ZF-based transceiver scheme that minimizes power without using subchannel selection has been proposed. In this section, a new power minimization using subchannel selection is presented. Firstly, assume that the encoding order of the users and the subchannel selection is fixed. Suppose the rate requirement for user k is R_k. Let η_k be the number of subchannels allocated to user k. Denote $N_D = \sum_{k=1}^{K} \eta_k$ as the total number of data streams. Then the SNR needed for each subchannel of user k is γ_k, where $\gamma_k = \left(2^{R_k/\eta_k} - 1\right)$. Denote $\mathbf{A} = \mathrm{blkd}(\mathbf{A}_1, \mathbf{A}_2, \ldots, \mathbf{A}_K)$ as the block diagonal receive equalization matrix, $\mathbf{F}$ as the transmit pre-equalization matrix, and $\mathbf{B}$ as the interference matrix. The problem of power minimization is formulated as

$$\begin{aligned} \text{minimize} \quad & \mathrm{Tr}(\mathbf{F}^H\mathbf{F}) \\ \text{subject to} \quad & \mathbf{A}\mathbf{H}\mathbf{F} = \sqrt{N_0}\boldsymbol{\Gamma}^{1/2}\mathbf{B} \\ & \mathbf{B} \in \mathbb{L} \ , \ \mathbf{A} \in \mathbb{B} \\ & \|\mathbf{A}(i,:)\| = 1 \qquad \text{for } 1 \leq i \leq N_D \ . \end{aligned} \tag{6}$$

where $\mathbb{L}$ is the set of all $N_D \times N_D$ lower triangular matrices with unit diagonal, $\mathbb{B}$ is the set of all block diagonal matrices such that each block is $\mathbf{A}_k \in \mathbb{C}^{\eta_k\times n_k}$. $\mathbf{F} \in \mathbb{C}^{N_T\times N_D}$, and $\boldsymbol{\Gamma} \in \mathbb{C}^{N_D\times N_D}$ is the diagonal matrix of SNR requirements. $\boldsymbol{\Gamma} = \mathrm{blkd}(\boldsymbol{\Gamma}_1, \ldots, \boldsymbol{\Gamma}_K)$, where $\boldsymbol{\Gamma}_k = \gamma_k\mathbf{I}_{\eta_k}$.

With subchannel selection for user k, $\mathbf{A}_k$ will not be square. Therefore $\mathbf{A}$ may not be unitary. The solution of [10] can be modified to this scenario. A reformulation of the BD-GMD with subchannel selection is defined:

$$\mathbf{P}^H\mathbf{H}\mathbf{Q} = \mathbf{L} \ ,$$

$$\begin{bmatrix} \mathbf{P}_1^H & \mathbf{0} \\ \mathbf{0} & \check{\mathbf{P}}_2^H \end{bmatrix} \begin{bmatrix} \mathbf{H}_1 \\ \check{\mathbf{H}}_2 \end{bmatrix} \begin{bmatrix} \mathbf{Q}_1 & \check{\mathbf{Q}}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{L}_1 & \mathbf{0} \\ \mathbf{X} & \check{\mathbf{L}}_2 \end{bmatrix} \ , \tag{7}$$

where $\mathbf{H}_1$ is the channel of user 1, and $\check{\mathbf{H}}_2$ contains the channels of user 2 onwards. $\mathbf{P}$ and $\mathbf{Q}$ are semi-unitary, i.e. $\mathbf{P}^H\mathbf{P} = \mathbf{I}$ and $\mathbf{Q}^H\mathbf{Q} = \mathbf{I}$. $\mathbf{L}$ is a lower triangular matrix which is block-equal-diagonal – the diagonal block corresponding to each particular user has equal diagonal elements.

Here, $\mathbf{L}_1$ can be obtained from $\mathbf{P}_1^H\mathbf{H}_1\mathbf{Q}_1 = \mathbf{L}_1$, which is the single-user GMD-SS. Since $\check{\mathbf{Q}}_2$ has to lie in the null space of $\mathbf{H}_1$, the projection matrix $(\mathbf{I} - \mathbf{Q}_1\mathbf{Q}_1^H)$ is used.

$$\check{\mathbf{P}}_2^H \left[\check{\mathbf{H}}_2 \left(\mathbf{I} - \mathbf{Q}_1\mathbf{Q}_1^H\right)\right] \check{\mathbf{Q}}_2 = \check{\mathbf{L}}_2 \ , \tag{8}$$

which is the same form as (7), so the algorithm proceeds recursively. Finally, $\mathbf{X}$ can be calculated as

$$\mathbf{X} = \check{\mathbf{P}}_2^H \check{\mathbf{H}}_2 \mathbf{Q}_1 \ . \quad (9)$$

Let the resulting equation (7) be called BD-GMD-SS.

Let $\mathbf{P}^H\mathbf{H}\mathbf{Q} = \mathbf{L}$ be the BD-GMD-SS for $\mathbf{H}$, and let $\mathbf{\Lambda} = \mathrm{diag}(\mathbf{L})$. $\mathbf{\Lambda} = \mathrm{blkd}(\mathbf{\Lambda}_1, \ldots, \mathbf{\Lambda}_K)$, where $\mathbf{\Lambda}_k = \lambda_k \mathbf{I}_{\eta_k}$ for some λ_k. Then, to solve (6), the following is applied [10]:

$$\begin{aligned} \mathbf{\Omega} &= \sqrt{N_0}\mathbf{\Gamma}^{1/2}\mathbf{\Lambda}^{-1} \ , \quad \mathbf{F} = \mathbf{Q}\mathbf{\Omega} \ , \\ \mathbf{B} &= \mathbf{\Omega}^{-1}\mathbf{\Lambda}^{-1}\mathbf{L}\mathbf{\Omega} \ , \quad \mathbf{A} = \mathbf{P}^H. \end{aligned} \quad (10)$$

V. Optimal User Ordering and Subchannel Selection

The optimal user ordering and subchannel selection can be found by an exhaustive search. However, the complexity here is much higher. In addition to searching through $K!$ orderings, all subchannel selection combinations have to be tested for each ordering. The BD-GMD-SS can be computed, to find the power, for each ordering and subchannel selection. The optimal case is chosen as the one that gives the minimum power.

To save on complexity, a ‘power-test’ version of the BD-GMD-SS can be used to find the transmit power, since that is the only relevant parameter of interest. As seen in section IV, $\mathbf{L}_1$ is obtained by the single-user GMD-SS. Therefore its diagonal elements are all equal to the geometric mean of the first η_k singular values, which can be obtained by a SVD. This gives $\mathbf{\Lambda}_1$. Next, as $\check{\mathbf{Q}}_2$ has to lie in the null space of $\mathbf{H}_1$, the projection matrix $(\mathbf{I} - \mathbf{V'}_1\mathbf{V'}_1^H)$ can be used, where the columns of $\mathbf{V'}_1$ are the first η_k right singular vectors of $\mathbf{H}_1$. Subsequently, the $\check{\mathbf{\Lambda}}_2$ can be found recursively, by applying the function on $\check{\mathbf{H}}_2(\mathbf{I} - \mathbf{V'}_1\mathbf{V'}_1^H)$. In this way, only K SVDs need to be carried out, instead of the complete BD-GMD.

Each user has at least one active subchannel, to satisfy its rate requirement. For each user ordering, the number of subchannel combinations to be tested is

$$N_c = \prod_{i=1}^{K} n_i \ . \quad (11)$$

The total number of tests would be $K!N_c$. This gives rise to a total of $KK!N_c$ SVDs.

VI. Efficient Method to Obtain User Ordering and Subchannel Selections

When the number of users K is large, the complexity would be reduced if only a subset of all $K!$ orderings are tested. In this section, an efficient method to obtain a suboptimal ordering is proposed. [10] describes 3 methods of ordering, assuming no subchannel selection. All three methods are non-iterative and do not involve convex optimization procedures. They select users in a “top-down” manner, from the first encoded user to the last encoded user. Method 1 is called successive closest match (SCM) which matches user SNR requirements with effective channel strengths after projection. Method 2 selects the user that gives the minimum λ_k. Method 3 selects the user that has the minimum channel strength $\mathrm{trace}(\mathbf{H}_k\mathbf{H}_k^H)/n_k$. A total of $K(K+1)$ determinant calculations are required to obtain all these orderings. For a detailed complexity analysis, the reader is referred to [10].

For each of these orderings, as well as the original unordered case, the optimal subchannel selection is evaluated with KN_c SVDs. Therefore $4KN_c$ SVDs would be performed for all 4 orderings, compared to $KK!N_c$ SVDs in section V. The ordering that gives the minimum power is chosen. Let this be called the ‘best choice ordering.’ This ordering is then used in the BD-GMD-SS to calculate the transmit and receive equalization matrices.

VII. Simulation Results

Consider the $8 \times [2, 2, 2, 2]$ downlink scenario. Let each user have 2 antennas. Let $\mathbf{R} = [\rho_1, \ldots, \rho_K]$ be the vector of rate requirements for each user. Let $\mathbf{c} = [c_1, \ldots, c_K]$ be the channel strengths of each user. The elements of the channel matrix of user k are modelled as i.i.d. zero-mean CSCG with variance c_k.

For the scenario of channel correlation, the following model is employed. Correlation between the channel responses is seen for the transmit antennas, as the base station is usually located in a high and unobstructed position [12]. The transmit correlation matrix for each user is dependent on the nominal angle of departure (AoD), $\bar{\theta}_k$, and the angular spread. As each user is located in rich local scattering vicinity, its antennas see uncorrelated channel responses. Similarly, there is no correlation between different users’ antennas, as they are usually far apart.

The channel for user k can be modelled as [14]

$$\mathbf{H}_k = \mathbf{H}_{w,k}(\mathbf{R}_{T,k}^{1/2})^T \ , \quad (12)$$

where $\mathbf{H}_{w,k} \in \mathbb{C}^{n_k \times N_T}$, the elements of which are i.i.d., zero-mean CSCG with unit variance, and $\mathbf{R}_{T,k} \in \mathbb{C}^{N_T \times N_T}$ is the transmit correlation matrix for user k. The matrix square root $(\cdot)^{1/2}$ is defined such that $\mathbf{R}^{1/2}\mathbf{R}^{1/2} = \mathbf{R}$.

To construct the covariance matrices [13], consider a uniform linear array (ULA) at the base station, where the antenna spacing is denoted as d. For an AoD θ, the steering vector is given by

$$\mathbf{a}(\theta) = [1, e^{j2\pi d \sin(\theta)/\varpi}, \cdots, e^{j2\pi(N_T-1)d\sin(\theta)/\varpi}]^T. \quad (13)$$

where ϖ is the carrier wavelength. Let the cell served by the base station be divided into S sectors. Then

$$\mathbf{R}_{T,k} = \int_{-\pi/S}^{\pi/S} \psi_k(\theta)\mathbf{a}(\theta)\mathbf{a}^H(\theta)d\theta, \quad (14)$$

where $\psi_k(\theta)$ is the ray-density function. The rays from the base station to each user are assumed to have a uniform density distribution. The nominal AoD is $\bar{\theta}_k$ and the angular spread is Δ_k. Thus

$$\psi_k(\theta) = \begin{cases} \frac{1}{\Delta_k} & \text{when } \bar{\theta}_k - \Delta_k/2 \le \theta \le \bar{\theta}_k + \Delta_k/2 \\ 0 & \text{otherwise.} \end{cases} \quad (15)$$

For all the figures, 'SS' means that BD-GMD with subchannel selection is used while 'no SS' means that subchannel selection is not used. 'u' denotes the unordered case, where the original user ordering is taken. 'no SS:opt' refers to the case where subchannel selection is not allowed but the optimal user ordering is found. The best subchannel selection for 'SS' is found by considering N_c cases. 'SS:u' denotes the unordered case with optimal subchannel selection applied. 'SS:opt' refers to the case where the optimal combination of user ordering and subchannel selection given by Section V is applied. For 'SS:bco,' the 'best choice ordering' described in Section VI is used. The subchannel selection is also optimal in this case. 'IB:opt' shows the power obtained by the optimal IB solution. Although not a DPC technique, the graph for ZF linear block diagonalization (LBD) [11] is shown, for the sake of comparison. Optimal water-filling is used for each user in the LBD scheme.

In Fig. 1, the transmit power is plotted against the rate requirement ρ, where the rate requirement for each user is ρ bps/Hz. It can be seen that a large improvement can be obtained when subchannel selection is allowed, even for uncorrelated channels. As the rate requirement increases, this gain reduces because more subchannels are used. Compared to the unordered schemes, ordering provides relatively small improvements. This is due to the similar channel strengths and similar rate requirements of all the users.

In Fig. 2, sum power versus target rate ρ is shown for the case with differentiated user rate requirements. The target rate is given by $\mathbf{R} = [\rho/2, 2\rho, \rho/2, 2\rho]$ bps/Hz. Here, user ordering plays a major role. In fact, the optimal ordering with no subchannel selection already performs better than the unordered case with optimal subchannel selection, for $\rho = 4$ and $\rho = 6$. This can be explained by the fact that different orderings result in different effective channel strengths of the users and proper ordering is required to match each user's rate requirement with its effective channel strength.

Fig. 3 shows the case of correlated channels and equal rate requirements. The nominal AoDs are set as $[-60^o, -20^o, 20^o, 60^o]$. The angular spread is set at 20^o for all users. Here the improvement from using subchannel selection is large, about 5dB for the unordered case. This is because in a rank deficient channel, the transmit power of each user can be reduced by choosing only a subset of available eigenchannels. In this case, optimal user ordering for the case of no subchannel selection is not able to compensate much for the effect due to correlation. This is reflected in the higher power of this method compared to the cases with subchannel selection allowed. This is due to the similar rate requirements of the users. Also, BD-GMD-SS has a transmit power around 0.5 dB higher than the optimal IB solution at $\rho = 4$ bps/Hz, whether or not user ordering is applied.

The effect of both differentiated rate requirements and correlated channels is plotted in Fig. 4. Similar to the previous figure, there is a substantial reduction in power when subchannel selection is allowed, about 5dB for the unordered case, due to the channel correlation. Furthermore, user ordering also provides a large benefit, as can be explained by the different target rates for different the users, which is also the phenomenum displayed in Fig. 2. For example, there is a power reduction of 4 dB for the case of no subchannel selection at $\rho = 4$. Also, the gap between the BD-GMD-SS scheme with optimal user ordering and subchannel selection and the optimal IB scheme is less than 0.5 dB at $\rho = 4$.

Fig. 5 illustrates the case where users have different channel strengths. This may be result of users being located at different distances from the base station. Even though the rate requirements are similar, there is a large improvement from ordering the users. This is because proper ordering matches the user rate requirements with the effective channel strengths after projection.

In all these graphs, it can be seen that by allowing subchannel selection, the transmit power can be reduced significantly. Optimal user ordering for the BD-GMD-SS scheme also improves the performance. It is able to provide a sum power close to the optimal IB solution but at a much lower complexity. Furthermore, the suboptimal method based on the 'best choice ordering' can be performed with even lesser computations without much loss in performance.

VIII. Conclusion

The optimal solution to the broadcast power minimization problem using DPC given user rate requirements has been solved optimally using iterative methods and convex optimization. However, these methods involve a large amount of computations. In this paper, BD-GMD-SS has been proposed for ZF power minimization. This ZF solution with the optimal ordering and subchannel selection can be found much faster than the optimal IB solution. Simulations have shown that the sum power obtained with the optimal BD-GMD-SS is not far from that of the optimal IB solution. A suboptimal method of ordering with further reduced complexity has also been proposed and has shown minimal performance loss.

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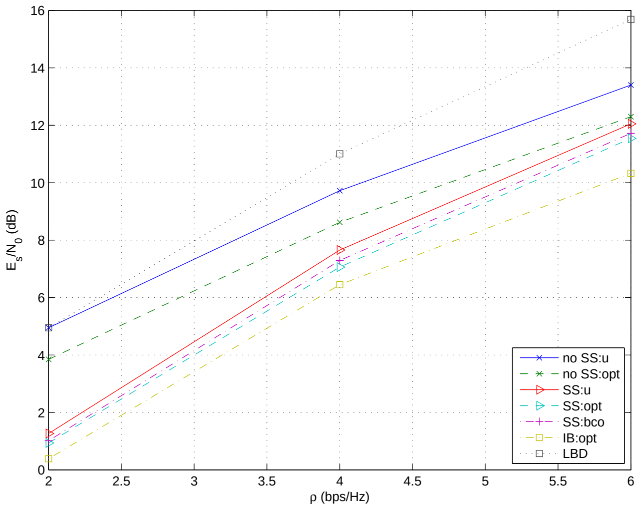

Fig. 1. $\mathbf{R} = [\rho, \rho, \rho, \rho]$. Uncorrelated channels.

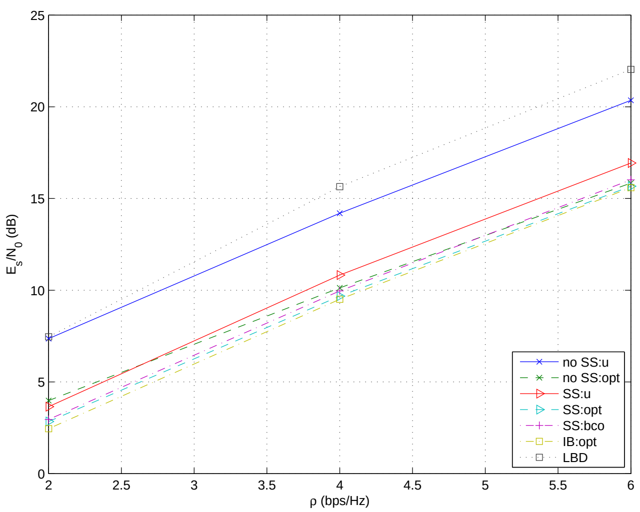

Fig. 2. $\mathbf{R} = [\rho/2, 2\rho, \rho/2, 2\rho]$. Uncorrelated channels.

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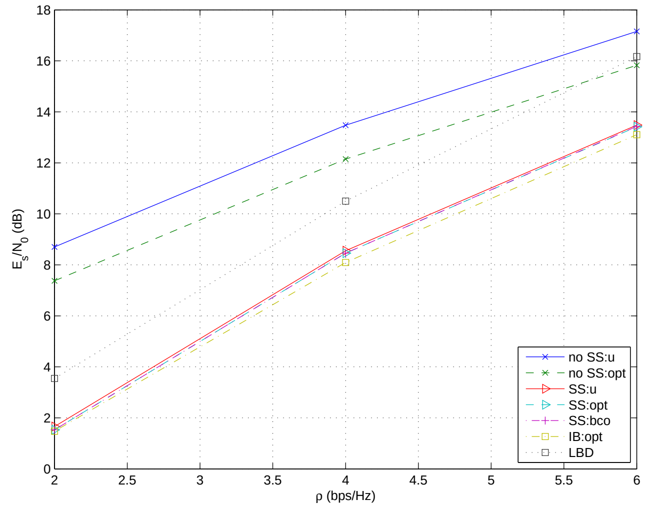

Fig. 3. $\mathbf{R} = [\rho, \rho, \rho, \rho]$. Correlated channels.

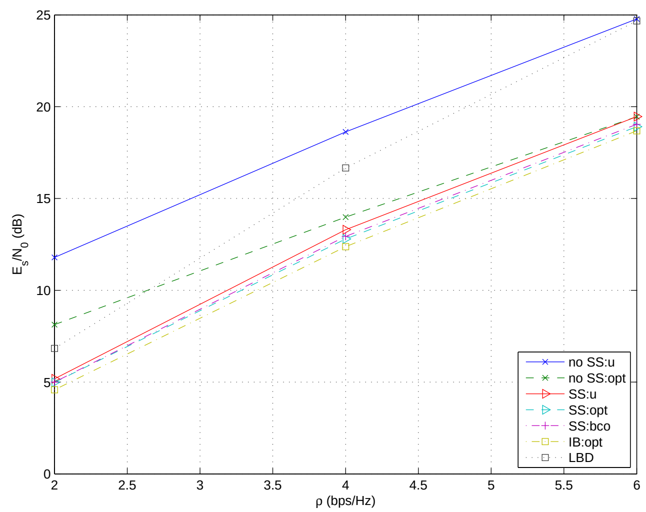

Fig. 4. $\mathbf{R} = [\rho/2, 2\rho, \rho/2, 2\rho]$. Correlated channels.

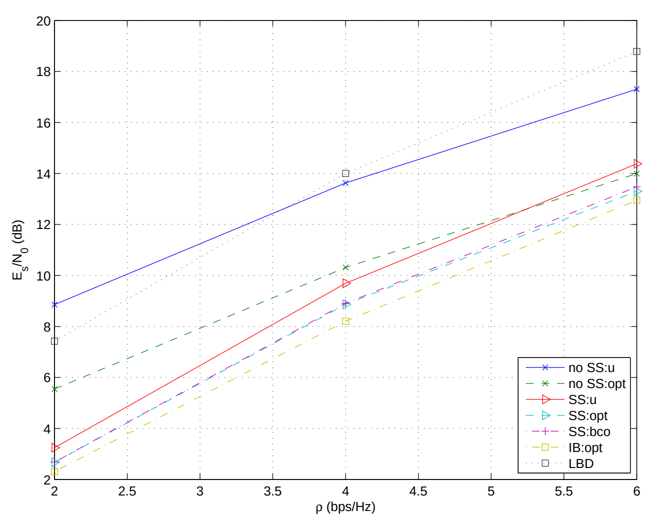

Fig. 5. $\mathbf{R} = [\rho, \rho, \rho, \rho]$. $\mathbf{c} = [1.5, 1.5, 0.5, 0.5]$. Uncorrelated channels.