MIMO Broadcast Comms using Block-Diagonal Uniform Channel Decomposition (BD-UCD)

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Overview

- Problem Statement
- Background
  - ➢ BD-GMD
  - ➢ MMSE-DFE
  - ➢ Uplink-Downlink Duality
- BD-UCD
- Equal Rate BD-UCD
- Simulations
- Conclusion

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Single-user MIMO

$$\mathbf{y} = \mathbf{Hx} + \mathbf{n}$$

- **Singular Value Decomposition (SVD)** $\mathbf{H} = \mathbf{USV}^H$
  - **Different constellation for each subchannel**
- **Geometric Mean Decomposition (GMD)[1]** $\mathbf{H} = \mathbf{QRP}^H$
  - **Same constellation for every subchannel (*Equal-Rate*)**
  - **Reduced transceiver complexity**
- **Uniform Channel Decomposition (UCD)[2]**
  - **MMSE-based *equal-rate* solution**
  - **Capacity-achieving**

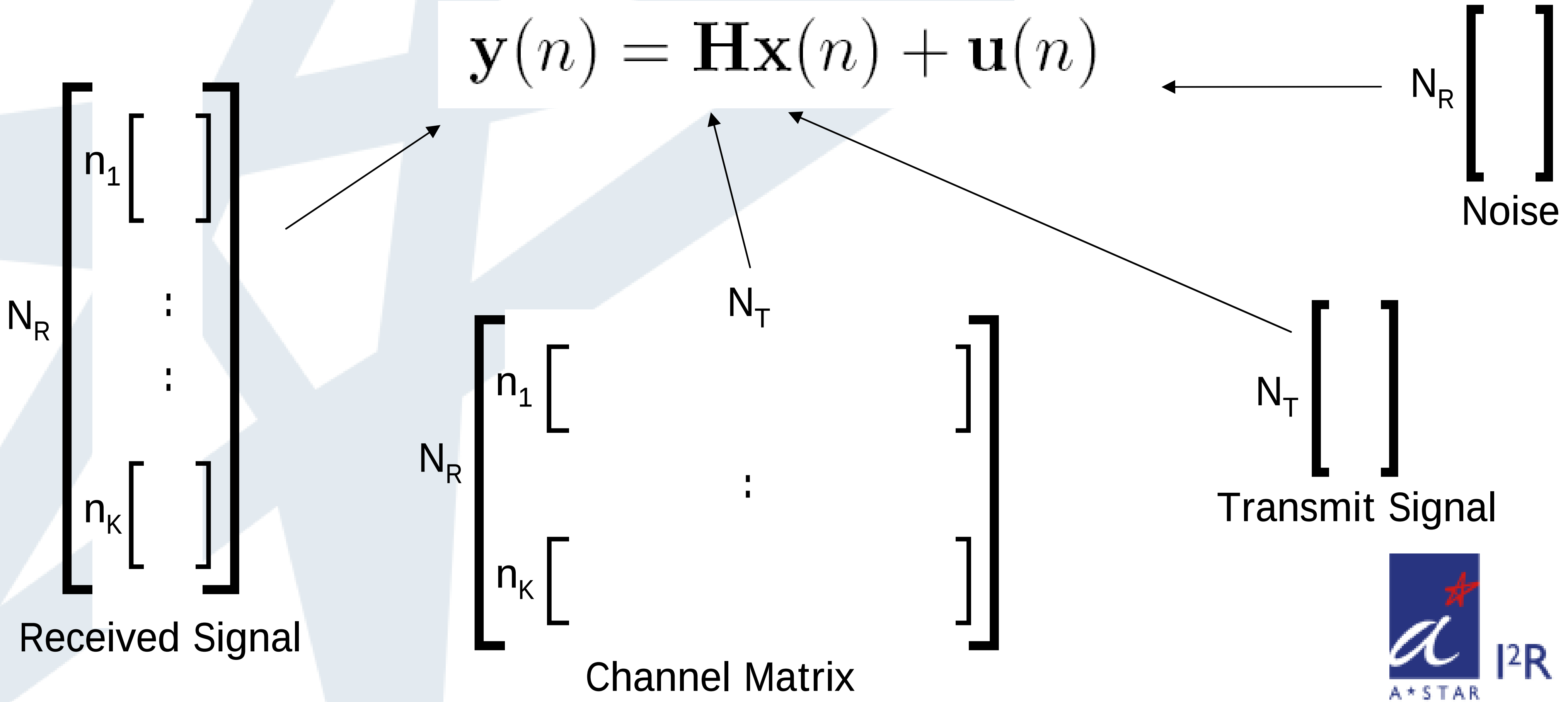
[1] Y. Jiang, J. Li and W. W. Hager, "Joint Transceiver Design for MIMO Communications Using Geometric Mean Decomposition," *IEEE Trans. Signal Processing*, vol. 53, no. 10, pp. 3791-3803, Oct. 2005.

[2] Y. Jiang, J. Li and W. Hager, "Uniform Channel Decomposition for MIMO Communications," IEEE Trans. Sig. Process., vol. 53, no. 11, pp. 4283-4294, Nov. 2005.

Multi-user MIMO

- N_T transmitting antennas
- K users with $n_1, \ldots, n_K$ receiving antennas
- $N_R = n_1 + \ldots + n_K$

$$\mathbf{y}(n) = \mathbf{H}\mathbf{x}(n) + \mathbf{u}(n)$$



Multi-user MIMO

- Receiving antennas do not all cooperate
- Block-Diagonal GMD (BD-GMD)[3]
  - ➢ Zero-forcing based solution
  - ➢ Dirty paper coding
  - ➢ Same constellation for subchannels of *each* user
  - ➢ Same constellation for *all* users if combined with optimized power control

[3] S. Lin, W. Ho and Y.-C. Liang, "Block-Diagonal Geometric Mean Decomposition (BD-GMD) for Multiuser MIMO Broadcast Channels," Proc. PIMRC, accepted for publication, Helsinki, Sep. 2006.

Problem Statement

	Single-user	Multi-user
Zero-Forcing based	GMD	BD-GMD
MMSE based	UCD	??

How do we design a MIMO multi-user MMSE-based scheme that:

- Is equal rate for each user,
- Is capacity achieving, and
- Uses dirty paper coding?

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Block-Diagonal GMD

$$H = P\,L\,Q^H$$

- P: Block Diagonal & Unitary
- L: Lower Triangular
- Q^H: Unitary

$$\begin{bmatrix} \mathbf{P}_1 & 0 & \dots & 0 \\ 0 & \mathbf{P}_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \mathbf{P}_K \end{bmatrix}$$

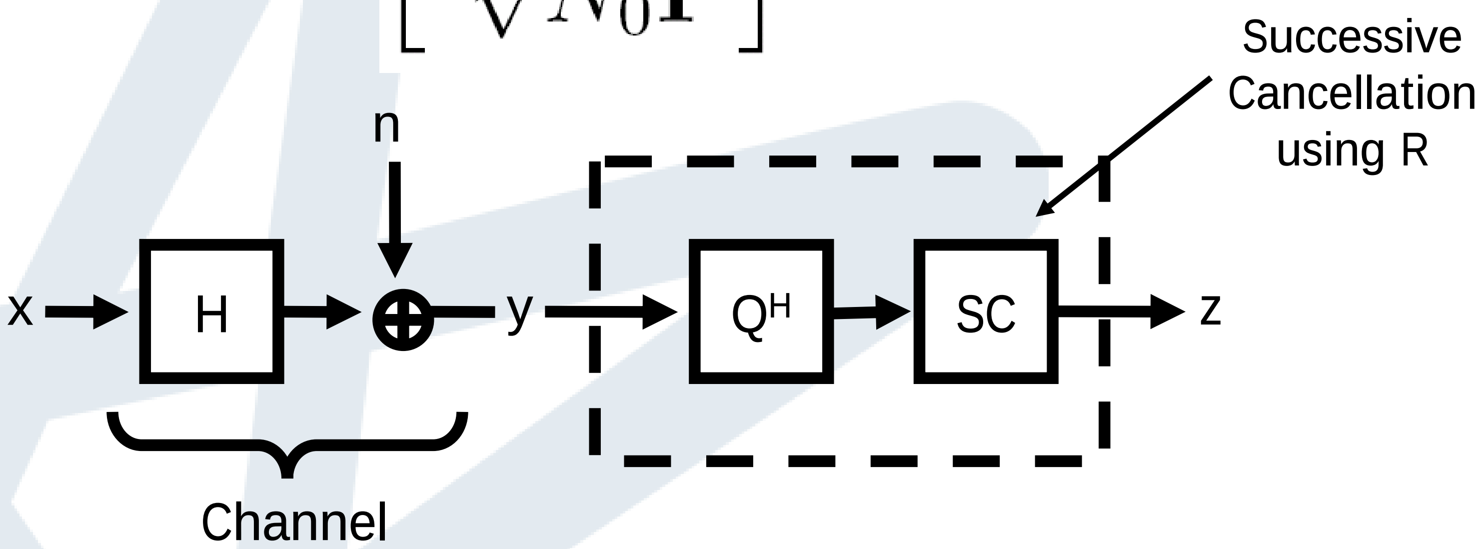
Each P_i is unitary.

$$\begin{bmatrix} \mathbf{L}_1 & 0 & \dots & 0 \\ X & \mathbf{L}_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ X & X & \dots & \mathbf{L}_K \end{bmatrix}$$

Each L_i is equal diagonal.

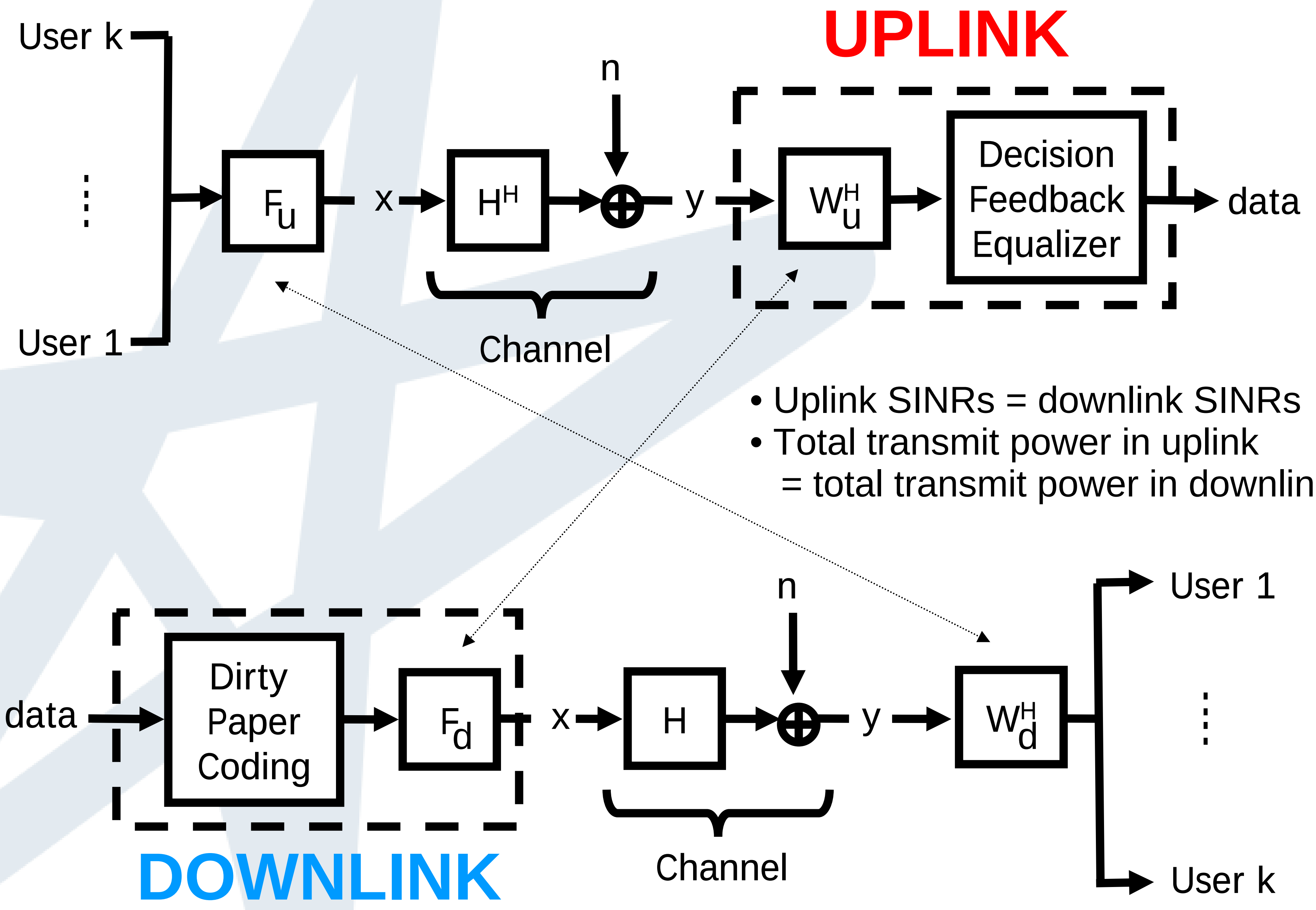
MMSE-DFE

$$\begin{bmatrix} \mathbf{H} \\ \sqrt{N_0}\mathbf{I} \end{bmatrix} = \mathbf{QR}$$



- Capacity achieving receiver technique

Uplink-Downlink Duality



- Uplink SINRs = downlink SINRs
- Total transmit power in uplink = total transmit power in downlink

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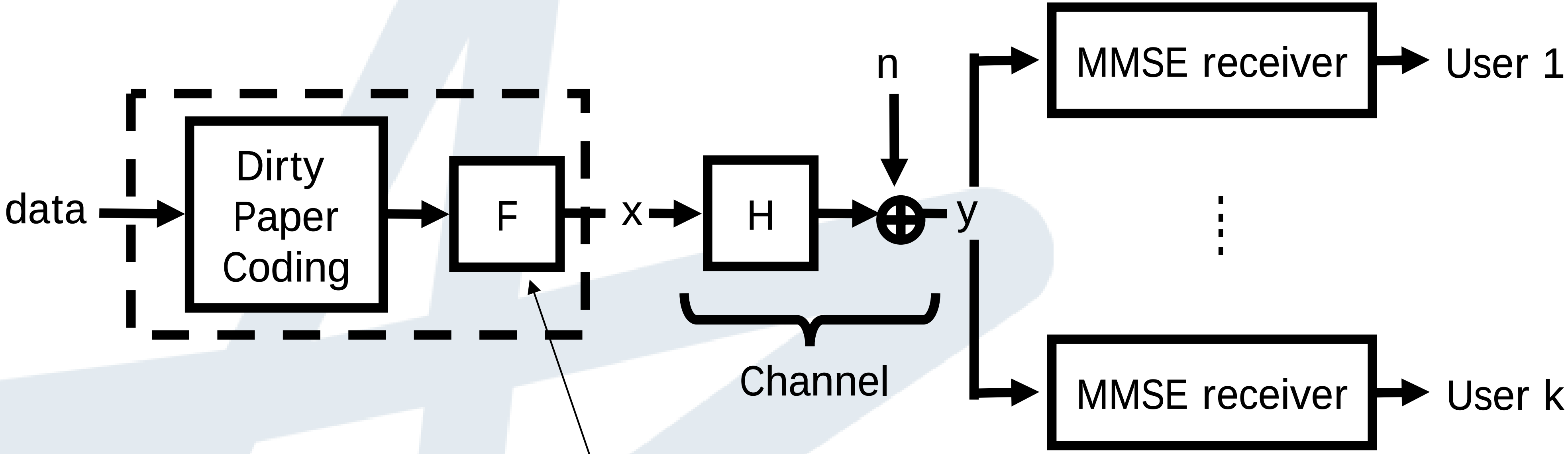
Problem Statement

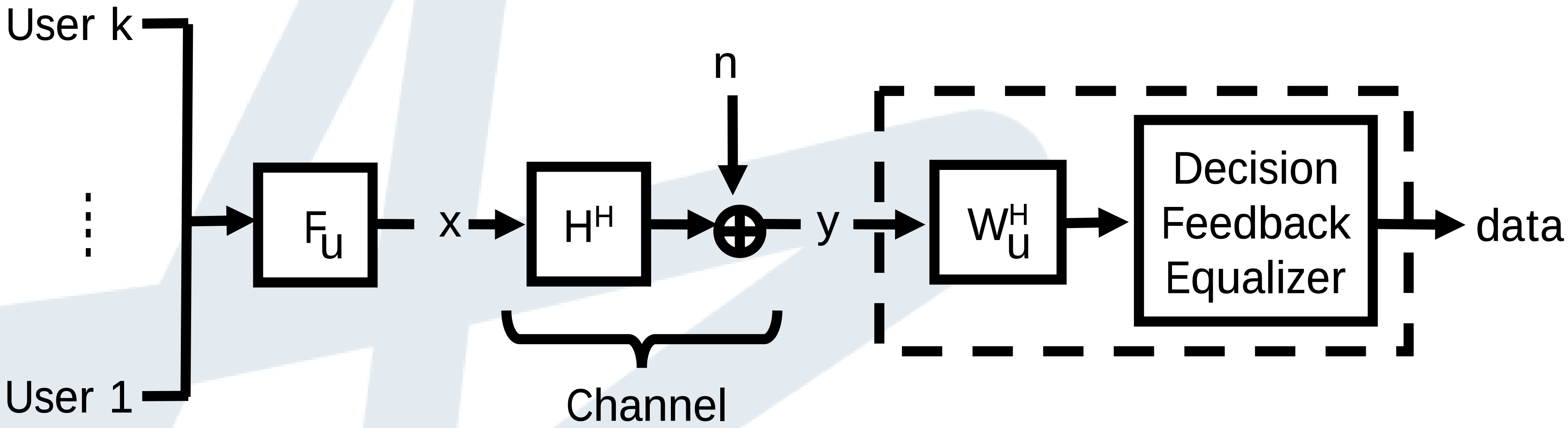


How do we design F (powerloading & beamforming) such that the scheme:

- Is equal rate for each user, and
- Is capacity achieving for a given transmit power constraint?

Strategy

- Consider dual uplink problem:



  - Find F_u such that each user enjoys equal-rates, and the scheme is capacity achieving

- Derive F from F_u and W_u using duality.

BD-UCD

- Let $\bar{F}$ be a precoder for the uplink channel that achieves the uplink sum-power capacity.
  - Sum-power iterative water-filling algorithm **[4]**
- Let $F_u = \bar{F}P$, where P is block-diagonal unitary.
  Find P such that in the MMSE-DFE derivation,

$$\begin{bmatrix} \mathbf{H}^H \bar{\mathbf{F}} \mathbf{P} \\ \sqrt{N_0}\mathbf{I} \end{bmatrix} = \mathbf{QR}$$

R is equal-diagonal in each diagonal block.

- To find P, use BD-GMD

$$\begin{bmatrix} \mathbf{H}^H \bar{\mathbf{F}} \\ \sqrt{N_0}\mathbf{I} \end{bmatrix}^H = \mathbf{PLQ}^H$$

[4] N. Jindal, W. Rhee, S. Vishwanath, S. A. Jafar and A. Goldsmith, ``Sum Power Iterative Water-Filling for Multi-Antenna Gaussian Broadcast Channels,'' *IEEE Trans. Inform. Theory*, vol. 51, no. 4, pp. 1570-1580, Apr. 2005.

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Equal-Rate BD-UCD

- Equal-rates for *ALL users!*
- *Same strategy as before:*
  - *by considering the dual uplink channel.*
- *Main problem is to find* $\bar{\mathbf{F}}$ *such that every user i enjoy the same rates* $R_i = R$,

$$R_i = \log \frac{\det\left(\mathbf{I} + \frac{1}{N_0}\sum_{j\le i} \mathbf{H}_j^H \bar{\mathbf{F}}_j \bar{\mathbf{F}}_j^H \mathbf{H}_j\right)}{\det\left(\mathbf{I} + \frac{1}{N_0}\sum_{j< i} \mathbf{H}_j^H \bar{\mathbf{F}}_j \bar{\mathbf{F}}_j^H \mathbf{H}_j\right)}$$

- *This can be solved near-optimally by an efficient iterative method (see paper for details.)*

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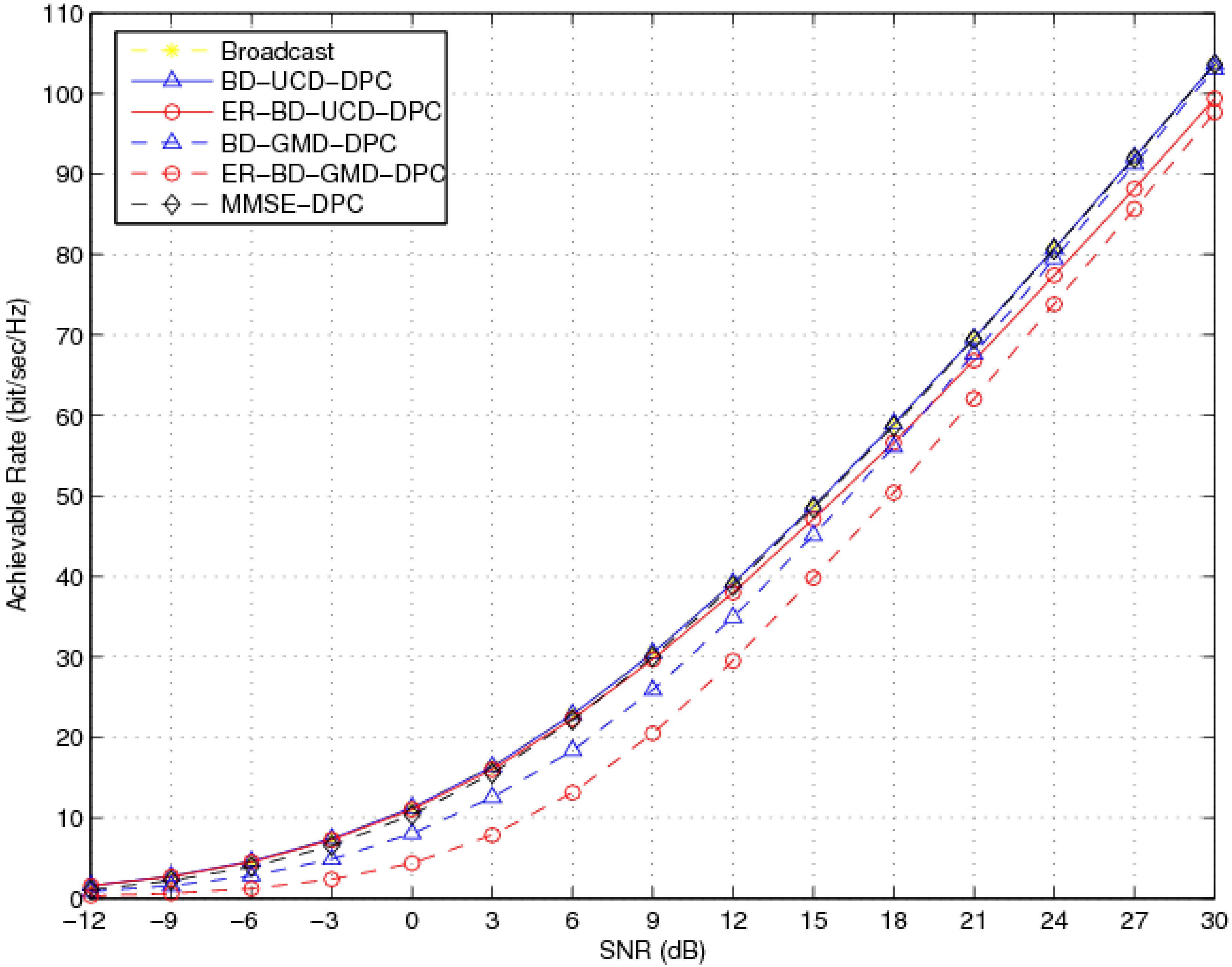
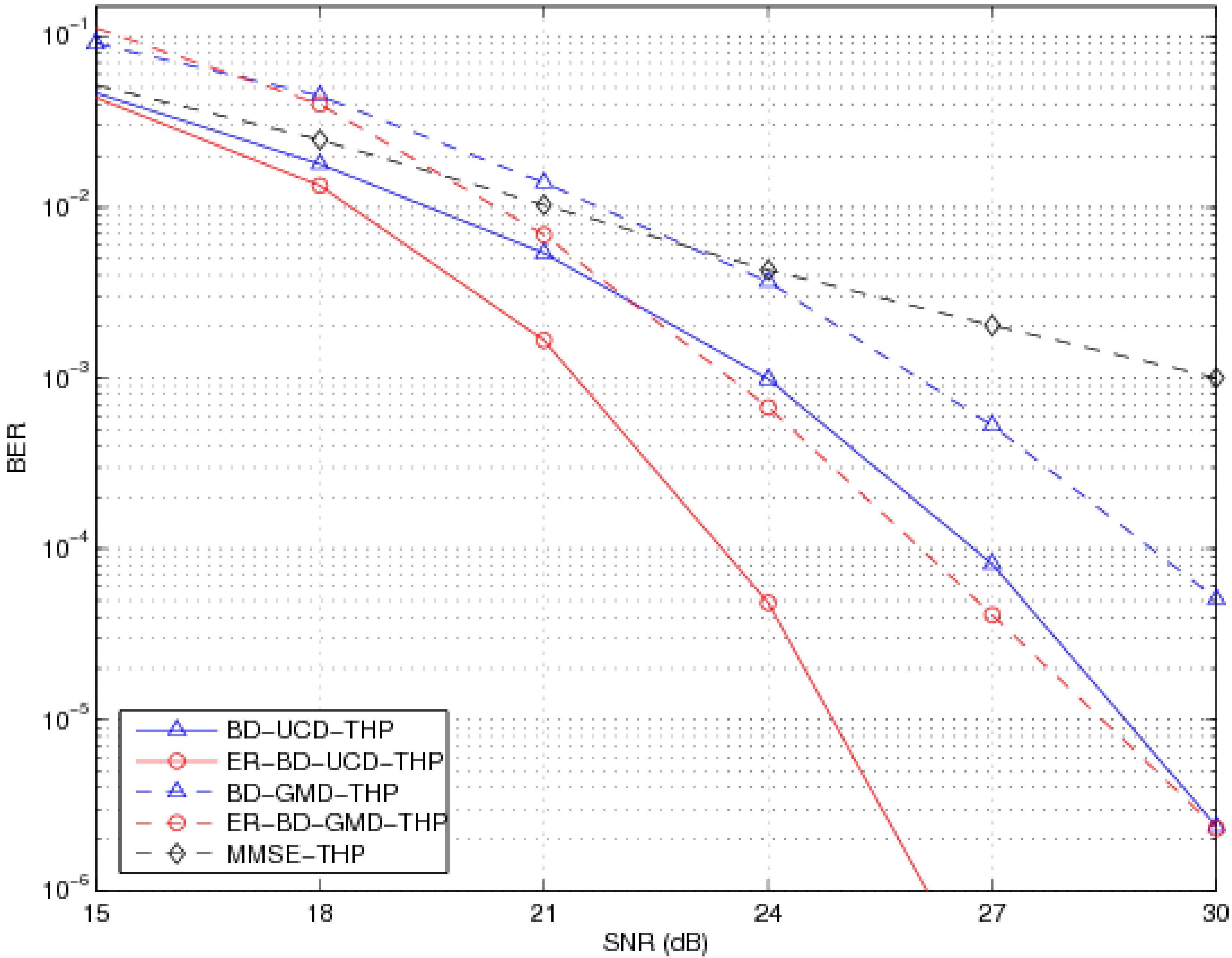
Comparison of Achievable Rates



Comparison of BERs



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Conclusion

- Using:
  - BD-GMD
  - Dirty Paper Coding
  - Uplink-Downlink Duality,
- We designed a BD-UCD scheme that is:
  - MMSE-based
  - Equal-rate for each user
  - Capacity-achieving.
- We also designed an ER-BD-UCD scheme via a near-optimal waterfilling algorithm.

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